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Semi-analytical solution for three-dimensional transient response of functionally graded annular plate on a two parameter viscoelastic foundation

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ABSTRACT

The three-dimensional transient analysis of functionally graded annular plates with arbitrary boundary conditions is carried out in this paper. The material properties of the FGM plate are assumed to vary smoothly in an exponential law along the thickness direction. The plate is assumed to rest on a two parameter viscoelastic foundation. A semi-analytical method, which integrates the state space method (SSM), Laplace transform and its inversion, as well as the one-dimensional differential quadrature method (DQM), is proposed to obtain the transient response of the plate. The state space method is used to obtain the analytical solution in the thickness direction. The differential quadrature method is employed to approximate the solution in the radial direction. The Laplace transform and the numerical inversion are used to obtain the solution in time domain. Numerical results show a good agreement between the response histories obtained by the present method and finite element method. The effects of the boundary conditions at the edges, the material graded index, the Winkler and shearing layer elastic coefficients, and the damping coefficient are studied. Numerical examples show that the peak response decreases as the material graded index, the Winkler and shearing layer elastic coefficients, and the damping coefficient increase. The results obtained in this paper can serve as benchmark data in further research.

1. Introduction

Functionally graded materials (FGMs) are inhomogeneous materials in which the mechanical or thermal properties vary in space gradually and smoothly [1,2]. More details about FGMs can be referred to Refs. [3,4]. With tailored design, generally, the material performance of FGMs can be superior to conventional composites [2,5]. Due to this fact, FGMs have become popular in many engineering applications, such as spacecraft industry, thermoelectric industry, power industry, human plants, and so on [2,4,6]. Motivated by the increasing application, FGMs have attracted a lot of research interest recently, especially in static and dynamic analysis [1,2,6–14].

The finite element method (FEM) is usually employed to investigate the statics and dynamics of FGM plate, due to its complicated fundamental equations [15–17]. The accuracy of FEM depends on mesh densities and time integration intervals, and the FEM cannot exactly reflect the smoothly varying elastic properties. Both the inappropriate discretization used for

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spatial derivation and time integration could result in spurious oscillations, thus affecting the accuracy of a numerical solution [18].

For this reason, many detailed studies using the classical plate theory or high-order shear deformation plate theories have been carried out. Reddy [19] investigated the axisymmetric bending and stretching of the FGM annular plate by employing the first-order shear deformation Mindlin plate theory. Cheng [20,21] derive field equations for a functionally graded plate whose deformations are governed by either the first-order shear deformation theory or the third-order shear deformation theory. Based on the classical plate theory and the Legendre interpolation method, Zhang [22] presented the static and quasi-static thermoviscoelastic analytical solutions for FGM thin plates. By adopting the classical generalized shear deformable theory, Lü [23] developed a generalized refined theory including surface effects for FGM ultra-thin films. Nevertheless, these theories always omit some displacement or stress variables and consistently give rise to numerical errors, which increase as thickness increases.

The state space method (SSM) is a three-dimensional elastic method in which all of the fundamental equations can be satisfied exactly, and all of the nine elastic constants can be accounted for [24]. Owing to its superior performance against various plate theories, this method has been widely used in order to investigate the statics and dynamics of FGM plates. Chen [6] approximated FGM shells by a laminated model, and then investigated the 3D free vibration of a FGM piezoelectric hollow cylinder filled with compressible fluid by using SSM. Lim [25,26] obtained an analytical solution for the temperature-dependent in-plane vibration of a simply supported FGM circular arch using two-dimensional SSM. Hosseini-Hashemi [27] investigated the free vibration of Levy-type thick FGM circular cylindrical shell panels by employing SSM. However, these studies were within the scope of statics or harmonic motions.

The transient problem has now been solved by employing the numerical inversion of Laplace transform to solve the complicated partial differential equations in time domain. Based on the numerical inversion method, the SSM and shooting method, Zhou [28] studied the three-dimensional transient thermoelastic responses of FGM rectangular plates with the material properties varying in an exponential law distribution along the thickness direction. Wen [5] derived three-dimensional solutions for the transient response of FGM rectangular plate, by using the numerical inversion method of Laplace transform developed by Durbin [29], the SSM and radial basis function method. Hasheminejad [30] similarly gave a three-dimensional elastodynamic solution for an arbitrary thick FGM rectangular plate resting on a two parameter viscoelastic foundation. The authors [31] proposed two novel algorithms for the numerical inversion of Laplace transform, and obtained semi-analytical solutions for the dynamic response of laminated plates subjected to underwater shock.

The above work has investigated the static, free vibration and transient responses of FGM plate in detail, but they are only for fully simply supported conditions. Owing to the limitation of traditional state space method, the differential quadrature method (DQM), a highly efficient numerical technique for obtaining numerical solutions of boundary/initial conditions [32], was introduced to the state space formalism by Chen [33] to derive semi-analytical elasticity solutions for the free vibration of laminated beams. The state space based differential quadrature method (SS-DQM) can treat precisely the edge boundary conditions at each discretized point [33,34]. Due to its superior characteristics, SS-DQM subsequently was used to derive semi-analytical solutions for the static and free vibration of FGM structures. Nie and Zhong [1,35] gave semi-analytical solutions for the three-dimensional vibrations of FGM circular/annular or sectorial circular/annular plates with material properties varying exponentially along the thickness direction. Alibeigloo [36] investigated the static response of FGM circular/annular plates embedded in piezoelectric layers. Kermani [37] analyzed the free vibration of multi-directional FGM circular/annular plates. However, the transient response is not involved in the research mentioned in this paragraph.

Annular plate has been widely applied in civil, mechanical, ocean and aerospace engineering. The statics and free vibration of FGM annular plates have, therefore, attracted a lot attention of many research groups, as mentioned above [38]. Hence, it is important to study the transient response of annular plate. But, to the authors' best knowledge, three-dimensional transient solutions for FGM annular plates under arbitrary boundary conditions have not yet been found.

In the present work, the transient response of FGM annular plate with arbitrary boundary conditions and resting on a two parameter viscoelastic foundation is investigated. The material properties are assumed to vary in an exponential distribution along the thickness direction. The fundamental equations are formulated by the state space method. A semi-analytical three-dimensional solution employing the DQM, Laplace transform and its numerical inversion is proposed. The solution is validated by comparing it with the results obtained by finite element method. The effects of boundary conditions, material graded index and foundation coefficients are evaluated.

2. Fundamental equations

2.1. Governing equations

Let us initially consider a linear elastic, and FGM annular plate of a constant thickness h, an inner radius b and outer radius a. The problem geometry and the cylindrical coordinate system are illustrated in Fig. 1, in which the (r,θ,z) frame is supposed to be located in space at the lower surface of the plate. Assuming the plate is transversely isotropic, the