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A novel method for crack detection in beam-like structures by measurements of natural frequencies

N.T. Khiem [*], L.K. Toan

Institute of mechanics, VAST, 264, Doi Can, Ba Dinh, Hanoi, Vietnam

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ABSTRACT

A novel method is proposed for calculating the natural frequencies of a multiple cracked beam and detecting unknown number of multiple cracks from the measured natural frequencies. First, an explicit expression of the natural frequencies through crack parameters is derived as a modification of the Rayleigh quotient for the multiple cracked beams that differ from the earlier ones by including nonlinear terms with respect to crack severity. This expression provides a simple tool for calculating the natural frequencies of the beam with arbitrary number of cracks instead of solving the complicated characteristic equation. The obtained nonlinear expression for natural frequencies in combination with the so-called crack scanning method proposed recently by the authors allowed the development of a novel procedure for consistent identification of unknown amount of cracks in the beam with a limited number of measured natural frequencies. The developed theory has been illustrated and validated by both numerical and experimental results.

1. Introduction

The condition assessment of structure and machinery is a vital concern in structural and mechanical engineering. A number of methods have been proposed to detect damage in structures and most of them are based on the change in their dynamic characteristics [1–3]. Among the numerous procedures developed for damage detection, the techniques based on damage-induced change in natural frequencies have been most early engaged [4–8] and they have been used until now [9–11]. This is because of the fact that the natural frequencies are most easily and accurately measured in comparison with other dynamic characteristics of a structure. The major drawbacks of the frequency-based approach are the weak sensitivity of the measured frequencies to damage, and the same change in frequencies might be caused by different damages. Also, the detection of unknown number of damages in a structure is in general an unsolved problem. Therefore, seeking the way to overcome the shortcomings of the frequency-based methods of damage detection is a promising subject.

The theoretical basis of the frequency-based methods for damage detection is the so-called characteristic equation that relates the natural frequencies to damage parameters. The first compact form of the characteristic equation was conducted in [12–15] for a beam-like structure with a single crack. Then, the equation has been established in different forms for the beam with multiple cracks [16–20]. Though the characteristic equation has been obtained explicitly, the natural frequencies could be computed just numerically as the implicit functions of damage parameters. This implicit representation of natural frequencies causes a difficulty in solving the problems associated with the damage detection from only natural frequencies.

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[*] Corresponding author.
E-mail addresses: ntkhiem@imech.ac.vn (N.T. Khiem), lktoan@imech.ac.vn (L.K. Toan).

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An explicit expression of natural frequencies in terms of crack magnitude was derived approximately in [21] and applied for damage detection in [22] for the case of small cracks by using the perturbation method. A system of linear equations relating the shift of natural frequencies with a variation of both the crack magnitudes and positions has been conducted in [23] but it was determined only numerically using the finite element method. By introducing the so-called element damage index, the authors of references [24–26] were able to express natural frequency shifts in terms of the damage indices in the form of linear equations that provide a useful tool for damage localization by measured natural frequencies. Although the conventional Rayleigh method was earlier employed for determining the natural frequency of a cracked beam by Shen and Pierre [27], an explicit expression of natural frequency was obtained much later by Fernandez-Saez et al. in [28] with the use of the Rayleigh method. Nevertheless, this appealing expression is applied only for calculating the fundamental frequency of a beam with a single crack. Later, Fernandez-Saez and Navarro [29] obtained a more accurate expression of natural frequencies for a singly cracked beam, but it was limited to applying for determining the upper and lower bounds of the fundamental frequencies only. Recently, an expansion of the Rayleigh quotient for calculating the natural frequencies of a cracked beam has been developed in [30] but as with the former results, it has not been straightforward to use for the crack detection problem.

Note here that though the comprehensive literature on the development of the frequency-based method for damage detection in structures has been published, very few papers are devoted to investigating the case of previously unknown number of damages. This posed a more widespread problem to predict also the number of damages mutually with their locations and sizes by the measurement of modal parameters. Developing the idea that emerged in the paper [8], the so-called crack scanning method is proposed in [31] to detect unknown number of cracks in a beam based on the measured natural mode shape. The main idea of the procedure is first to estimate unknown magnitudes of all the cracks assumed at a chosen grid of positions in the structure using the given data and this process is performed iteratively by eliminating the positions from the grid where the estimated magnitudes are zero or negative. The actual cracks would be acknowledged at the locations of the grid that could not be reduced by the removing positions with zero and negative magnitude. Actually, the proposed procedure enables us to determine not only the location and magnitude but also the quantity of cracks. Certainly, this procedure can be applied not only for the case of measured mode shapes as performed in [31] but also for the case of other modal parameters such as natural frequencies or frequency response functions.

The present paper aims to develop the scanning method for detecting unknown number of cracks in a beam by the measurement of natural frequencies. Firstly, the Rayleigh quotient is derived for the multiple cracked beam that enables us to conduct an explicit expression of natural frequencies in terms of crack positions and sizes by choosing the shape function first suggested in [28]. Such obtained frequency representation provides a simple and efficient tool for calculating every natural frequency of the beam with arbitrary number of cracks. The most important difference of the constructed explicit expression from those derived by the perturbation [21], sensitivity method [23] and the energy approach [24–26] is that the obtained herein expression included additionally the nonlinear terms of the crack magnitudes. Then, the obtained explicit expression is straightforward to apply the aforementioned scanning method for identification of the multiple cracks from natural frequencies. In this regard, the nonlinear terms taken into account could be helpful for us to overcome the non-uniqueness solution of damage detection problem in the beam with symmetrical boundary conditions. The theoretical development is validated by both numerical and experimental examples.

2. The Rayleigh quotient for the multiple cracked beam

Let's consider a uniform Euler–Bernoulli beam with clamped ends and the following material and geometrical constants: Young's modulus E, mass density ρ, length L, cross section area $F=b\times h$ and moment of inertia I. Suppose, moreover, that the beam has been damaged to crack at a number of positions $0\prec e_1\prec \ldots \prec e_n \prec 1$ with the depth $(a_1,\ldots,\ a_n)$. If the spring model of the cracks is adopted the spring stiffness K_j is calculated from the crack depth a_j by [16]

$$\gamma_j = EI/LK_j = (5.346h/L)I_c(a_j/h); \tag{1}$$

$$I_c(z) = 1.8624z^2 - 3.95z^3 + 16.375z^4 - 37.226z^5 + 76.81z^6 - 126.9z^7 + 172z^8 - 143.97z^9 + 66.56z^{10},$$

where the parameter $\gamma_j=EI/LK_j$ has been introduced to represent severity of the crack and termed by crack magnitude.

For the beam, kth natural frequency and mode shape denoted by ω_k, $\phi_k(x)$ satisfy equation

$$d^4\phi_k(x)/dx^4 - \lambda_k^4\phi_k(x) = 0;\ x\in(e_{j-1},e_j),\quad j=1,\ldots,n+1;\ e_0=0,\ e_{n+1}=1;$$
$$\lambda_k^4 = L^4\rho F\omega_k^2/EI \tag{2}$$

and boundary conditions

$$\phi_k(0)=\phi_k'(0)=\phi_k(1)=\phi_k'(1)=0. \tag{3}$$

Additionally, the mode shape $\phi_k(x)$ should satisfy the following conditions at cracks:

$$\phi_k(e_j^-)=\phi_k(e_j^+);\ \phi_k''(e_j^-)=\phi_k''(e_j^+);\ \phi_k'''(e_j^-)=\phi_k'''(e_j^+);\ [\phi_k'(e_j^+)-\phi_k(e_j^-)]=\gamma_j\phi_k''(e_j),\ j=1,\ldots,n. \tag{4}$$