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Some analytical expressions to measure the accuracy of the "equivalent von Mises stress" in vibration multiaxial fatigue

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ABSTRACT

The "equivalent von Mises stress" (EVMS) was first proposed in 1994 by Preumont and co-workers as a frequency domain reformulation of von Mises stress, for the fatigue analysis of vibrating structures under multiaxial random stresses. The EVMS criterion is a simple, but very powerful tool to estimate fatigue damage with time domain analysis of simulated stress histories, or frequency domain evaluation by spectral methods. Despite its simplicity, the EVMS criterion is based on some inherent assumptions, which may lead to inaccurate damage estimations in some particular conditions (e.g. materials with very different axial/bending and torsion S–N curves). This paper aims to derive some analytical expressions to measure the accuracy of EVMS criterion for various combinations of material fatigue properties and loading conditions (e.g. combined axial/bending and torsion loadings). These expressions constitute an original contribution, as similar analytical approaches have not been proposed in literature. The accuracy of EVMS approach is then tested with typical material fatigue properties from literature. The range of applicability of EVMS criterion is then be identified for specified intervals and combinations of S–N parameters.

1. Introduction

Metallic structures excited by random loadings or vibrations are exposed to high-cycle fatigue caused by random stresses. Even in the simplest case, the local state of stress is generally multiaxial (i.e. two or more stress components). Appropriate fatigue criteria must then be used to correctly assess the structural durability under such complex multiaxial stress states [1].

Multiaxial fatigue criteria have historically been developed as time domain approaches, i.e. based on step-by-step algorithms applied to simulated or measured stress time-histories. Progressively, they have been reformulated also in the frequency domain by a power spectral density (PSD) characterisation of multiaxial random stresses. The frequency domain approach allows a simple description of the relevant statistical properties of multiaxial random stresses. It allows a quick evaluation of structure fatigue damage by directly processing stress histories simulated in time domain, or by using analytical expressions given by multiaxial spectral methods defined in frequency domain. Compared to time domain approach, the frequency domain approach can then take advantage of structural dynamics for the fatigue analysis of linear structures under random vibrations.

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Nomenclature

c_σ, c_τ	fatigue strength coefficient of S–N curve for normal and shear stress
d_{NB}, d_{WB}	damage intensity (damage/s) for narrow-band and wide-band stress
$e_d=h(\nu_r)$	damage index at variance ratio ν_r for normal and shear stress
k_σ, k_τ	exponents of S–N curves
N	number of cycles to failure
N_A	reference number of cycles (typically $N_A=2\times10^6$)
$\mathbf{Q}$	matrix with coefficient of von Mises stress
r_d	damage index of EVMS criterion with random shear stress
$r_k=k_\tau/k_\sigma$	ratio of S–N exponents
$r_{\sigma\tau}=\sigma_A/\tau_A$	ratio of fatigue strength amplitudes
$S(f)$	double-sided PSD
$S_{eq}(f)$	double-sided PSD of the EVMS
$\mathbf{S}(f)$	double-sided PSD matrix
V_σ, V_τ	variance of normal and shear stress
$\nu_r=V_\tau/V_\sigma$	variance ratio
α_1, α_2	bandwidth parameters
η	bandwidth correction factor
η_{eq}	bandwidth correction factor for $S_{eq}(f)$
λ_n	spectral moment of order n for $S(f)$
$\lambda_{n,\ eq}$	spectral moment of order n for $S_{eq}(f)$
$\lambda_{n,\ hk}$	spectral moment of order n for $S_{hk}(f)$
ν_0	frequency of mean upcrossings
$\nu_{0,eq}$	frequency of mean upcrossings of $\sigma_{eq}(t)$
σ_A, τ_A	fatigue amplitude strength for normal and shear stress
$\sigma(t)$, $\tau(t)$	normal and shear stress
$\sigma_{eq}(t)$	equivalent von Mises stress (EVMS)
$\sigma_{vm}(t)$	von Mises stress

Several multiaxial fatigue criteria, originally developed in time domain, have next been reformulated also in frequency domain as multiaxial spectral methods. These criteria can be used to quickly estimate the fatigue damage in frequency domain. Some are based on a suitable equivalent uniaxial stress, as the "equivalent von Mises stress" [2–7] or the shear/normal stress on the critical plane in critical plane multiaxial criteria [8,9]. A frequency domain approach was also devised to identify the shear stress amplitude of a random stress path in critical plane or invariants-based multiaxial criteria [10]. The literature also accounts for a stress invariants-based criterion formulated in the frequency domain [11,12]. Finally, interesting analogies between uniaxial and multiaxial fatigue criteria defined in frequency domain have recently been pointed out in [13].

The interest is focused here on the "equivalent von Mises stress" (EVMS) criterion, which was first proposed in 1994 by Preumont and co-workers as a frequency domain reformulation of the von Mises stress [2]. The EVMS is a uniaxial random stress that is characterised in frequency domain by a PSD, which is calculated as a linear combination of the auto- and cross-spectra of multiaxial random stresses. The EVMS criterion by Preumont et al. is based on the assumption that the "equivalent von Mises stress" gives the same damage value as the one caused by a multiaxial random stress. The damage caused by the EVMS can easily be calculated by time domain processing of simulated uniaxial time-histories or, more efficiently, it can be estimated by frequency domain analysis with uniaxial spectral methods. In both cases, the EVMS criterion provides an efficient approach that greatly simplifies the fatigue damage assessment in vibration multiaxial fatigue.

Thanks to its simplicity and its ease of use, the EVMS method has largely been applied in both academic and applied research to vibration fatigue problems. For example, Pitoiset et al. [3,4] reported extensive simulations and comparisons of fatigue life determined in time and frequency domains. Application to the fatigue life assessment of the nozzle of the Vulcan motor working in the Arianne V rocket has also been investigated [5]. Other applications of the EVMS criterion can be found in [14–16]. Furthermore, the theoretical framework of EVMS criterion was also used to reformulate in the frequency domain other multiaxial fatigue criteria (e.g. multiaxial rainflow counting, stress invariants-based Crossland and Sines criteria) [6,7,17,18].

Undoubtedly, the EVMS criterion can be recognised as a fundamental achievement in multiaxial vibration fatigue. Despite this, a closer look would reveal that the EVMS criterion is based on some intrinsic assumptions, which may lead to incorrect results in some circumstances (e.g. different S–N fatigue curves for normal and shear stress). Some studies in literature [7,12,19,20] have already pointed out the assumptions of EVMS criterion: the S–N lines for normal and shear stress must have equal slopes, the fatigue strengths must be scaled by a factor $\sqrt{3}$, which appears in the definition of von Mises stress. Such assumptions are severe limitations to obtain a correct fatigue damage assessment for other combinations of materials fatigue properties.

Unfortunately, the studies existing in literature did not investigate the accuracy of EVMS criterion when its assumptions are not satisfied. This lack of knowledge makes then uncertain the evaluation of the range of applicability of EVMS criterion and its degree of reliability.

With the aim to shed light on the performance of EVMS criterion, this paper presents a critical analysis of the EVMS criterion by Preumont et al. Unlike previous studies in literature, this paper derives some analytical expressions to directly assess the accuracy of fatigue damage estimations by EVMS criterion for various combinations of material fatigue properties. The simple case of uniaxial normal and shear random stresses is analysed first. An analytical expression to measure the accuracy of EVMS method with pure shear stress is derived. This expression is then used to devise an interpolating function to assess the damage accuracy of EVMS approach for a biaxial random stress (e.g. bending plus torsion random loading) as