Journal of Sound and Vibration 333 (2014) 4376–4401

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Numerical and experimental analysis of uncertainty on modal parameters estimated with the stochastic subspace method

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ARTICLE INFO

Article history:
Received 9 October 2013
Received in revised form
15 April 2014
Accepted 18 April 2014
Handling Editor: K. Shin
Available online 13 May 2014

ABSTRACT

Modal parameters of structures are often used as inputs for finite element model updating, vibration control, structural design or structural health monitoring (SHM). In order to test the robustness of these methods, it is a common practice to introduce uncertainty on the eigenfrequencies and modal damping coefficients under the form of a Gaussian perturbation, while the uncertainty on the mode shapes is modeled in the form of independent Gaussian noise at each measured location. A more rigorous approach consists however in adding uncorrelated noise on the time domain responses at each sensor before proceeding to an operational modal analysis. In this paper, we study in detail the resulting uncertainty when modal analysis is performed using the stochastic subspace identification method. A Monte-Carlo simulation is performed on a simply supported beam, and the uncertainty on a set of 5000 modal parameters identified with the stochastic subspace identification method is discussed. Next, 4000 experimental modal identifications of a small clamped–free steel plate equipped with 8 piezoelectric patches are performed in order to confirm the conclusions drawn in the numerical case study. In particular, the results point out that the uncertainty on eigenfrequencies and modal damping coefficients may exhibit a non-normal distribution, and that there is a non-negligible spatial correlation between the uncertainty on mode shapes at sensors of different locations.

1. Introduction

Further to huge advances in the development of operational modal analysis (OMA) tools [1,2], there is an increasing interest in the estimation of modal parameters from vibration responses for different purposes. Finite element model updating which consists in adapting a numerical model of the structure of interest is a field in which modal parameters are extensively used. Typically, finite element model updating aims at improving the numerical models by minimizing the distance between the measured data and the model by modifying the numerical model. While any dynamic signature can be considered for this purpose (time histories, frequency responses or energies for instance), the use of mode shapes and eigenfrequencies is probably the most common approach [3–5].

Methods considering modal parameters for damage assessment have been widely studied in the last few decades. The techniques for structural health monitoring (SHM) can be classified into two big families, depending on the need or not of a numerical physical model (finite element model) of the structure of interest. Doebling et al. propose a detailed overview of

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http://dx.doi.org/10.1016/j.jsv.2014.04.039
0022-460X/

the first category of methods based on updating modal parameters in [6]. While model-based methods require a numerical model of the structure to identify the damage, the second category of methods which is referred to as data-based methods rely only on measured data to identify damage. While time domain responses of sensors can be used for damage assessment by considering guided waves [7], or autoregressive models [8–10], methods using frequency responses and modal parameters are much more studied. The use of eigenfrequencies has been widely investigated for damage assessment, since cracks typically decreases the eigenfrequencies. Salawu provides a wide overview of the methods using changes of eigenfrequencies for damage assessment in [11]. Modal damping coefficients have also been considered for structural health monitoring: cracks create new surfaces increasing the dissipation energy in most cases [12], but these parameters are more sensitive to the test procedure as well as to the analysis methods than eigenfrequencies, making them less reliable [13]. While eigenfrequencies and modal damping coefficients are global structural parameters, mode shapes are locally affected by a damage, which makes them particularly interesting to locate the damages. The most straightforward way to proceed consists in comparing directly the undamaged and the damaged mode shapes. The idea to locate damage from curvature mode shapes has been proposed by Pandey in [14], since it has been observed that the effect of damage is located in the close vicinity of the damage. Other methods use approximations of the stiffness and flexibility matrices [15] from the identified mode shapes and eigenfrequencies in order to locate damages. Stubbs and Kim have also proposed in [16] to take advantage of the decrease of strain modal energy in beam-like structures to identify the position of damages, and the technique has been extended to plate-like structures in [17].

Because noise is always present in measurements, and that operational modal analysis introduces some uncertainty in the identified modal parameters [18], the validation of the previous methods calls for a study of their efficiency when some uncertainty is taken into account in the modal parameters of interest. Traditionally, when testing the robustness of such types of methods, numerical models are used in order to compute the modal parameters which will be used as input measurements, and noise is added directly on the modal parameters. Most of the time, robustness studies consider uncertainties on the mode shapes in the form of independent white noise added at each measured location such as in [19–21]. Similarly, the uncertainty on the identified eigenfrequencies and modal damping coefficients consist very often in perturbing the reference eigenfrequency or modal damping with a white noise (Gaussian noise), as in [22]. Other research studies consider noise added directly on the time domain sensor response which is more realistic, but the methods developed are mainly using time domain sensor responses, as in [23,24].

Several studies which deal with the estimation of the uncertainty on modal parameters from a single stochastic subspace identification can be found in the literature [18,25,26]. Using a perturbation analysis, these techniques are interesting to estimate the variance of identified eigenfrequencies and modal damping coefficients as well as the covariance of the identified mode shapes, and are usually validated with a Monte-Carlo simulation which is much more computationally expensive, but which gives a better estimation of the (co)variances. However, Carden and Mita pointed out in [27] that the modal parameters may exhibit non-normal distribution, and that the variances are not adequate to estimate the confidence intervals in that case. The aim of the present paper is to study in detail the effect of noise measurement on the uncertainty of modal parameters obtained with stochastic subspace identification [1]. A Monte-Carlo simulation on a numerical case study and an experimental validation are performed. These results are used to assess the uncertainty on the modal parameters obtained with successive modal identifications, and a comparison is performed when the uncertainty is directly added on the modal parameters obtained with a unique modal identification.

This paper is organized as follows: Section 2 deals with a numerical study of the uncertainty on modal parameters due to measurement noise. The structure investigated is a simply supported beam equipped with 11 equally distributed strain sensors. 5000 samples of the dynamic response of the beam excited by a band-limited white noise signal are computed, and the modal properties are identified for each sample. Noise measurement is added directly on the sensor responses before the modal identification, and we compare the uncertainty obtained on eigenfrequencies, modal damping coefficients and mode shapes with the classic approach in which the uncertainty is modeled as a Gaussian noise added directly on the modal parameters. In particular, the correlation matrices are computed for each mode shape of interest and show that the classic approach for which the noise of the mode shapes projected on the sensors is added independently at each sensor location neglects the spatial correlation which exists between the noise of sensors at different locations. Section 3 investigates the experimental uncertainty of modal parameters on a small clamped–free steel plate equipped with 8 piezoelectric patches and excited with piezoceramic patch. The same analysis as in the numerical case study is performed, and the experimental results confirm the main numerical observations which are: (i) the spatial correlation in the noise of mode shapes between sensors at different locations, and (ii) the non-normal distribution of the identified eigenfrequencies and modal damping coefficients. These observations illustrate very well that the robustness with respect to noise measurement of any method using modal parameters should be based on Monte-Carlo simulations in which the noise is introduced on the sensor responses in time domain, before the modal identification.

2. Numerical study of the uncertainty on modal parameters due to measurement noise

2.1. Description of the case study

The numerical case study deals with a 1 m × 0.1 m × 0.1 m simply supported beam made of concrete (Fig. 1) that has already been investigated [28]. The beam is modeled with 100 Euler–Bernoulli beam elements using the *Structural Dynamics*