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A bifurcation-based coupled linear-bistable system for microscale mass sensing

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ABSTRACT

Bifurcation-based mass sensing provides for dramatically enhanced detection sensitivity and less performance deterioration due to measurement noise as compared to frequency shift-based methods. Recent bifurcation-based mass sensing studies have employed directly excited nonlinear oscillators to induce critical jump events, but the approaches may still require active tracking hardware to determine exact mass adsorption, could induce adverse nonlinear phenomena by prolonged excitation near the bifurcation, and are limited in the number and versatility of jump events. In this work, an alternative sensor architecture and method for mass sensing are presented to address these concerns. The architecture is based upon the coupling of a host linear structure to a small bistable inclusion. It is shown that the sensor enables unique functionality including means for passive mass quantification and direct adjustment of bifurcation sweeping rate for reliable detection and enhanced robustness to noise. Deterministic, stochastic, and non-stationary analyses demonstrate the operational principles and sensitivities of the method while experiments with proof-of-concept samples corroborate analytical results and give clear evidence of the advantages of the new approach.

1. Introduction

Micro/nanoelectromechanical systems (M/NEMS) have proliferated as low-cost and high-performance sensors across a broad range of applications, including magnetic field detection, gyroscopic orientation, and biological, chemical, and atomic mass monitoring [1,2]. The latter field has received dramatic research attention due to analytical and experimental demonstration that these systems may probe to quantum scales, indicating that mass/force detection ability of micro- and nanomechanical sensors is useful for day-to-day applications (e.g. mercury detection [3]) as well as for fundamental physics exploration [4–6].

A common protocol for microscale mass sensing is a derived translation between mass adsorbed upon the resonating sensor and a detectable change in the fundamental mode natural frequency [1–6]. Although this is a useful approach, researchers continue to tackle with the inherently nonlinear dynamics of resonators on these scales which makes an expression between accumulated mass and natural frequency shift less straightforward than ideally obtained with linearity [1]. While some have investigated methods to offset one nonlinearity by another (e.g. introducing softening Duffing effects to counteract hardening Duffing behaviors [7,8]), recent research has developed alternative sensing approaches based upon the inherent nonlinear dynamics of MEMS. These latter studies harness nonlinear bifurcation phenomena which serve as dramatic

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amplitude-based sensing measures, following a methodology earlier developed for the Josephson junction or bifurcation amplifier [9].

One significant benefit of bifurcation-based sensing is that the metric of change is an unmistakable jump or drop in sensor response amplitude once the bifurcation is crossed. The techniques are much less susceptible to damping than frequency shift-based approaches because bifurcations may be activated regardless of damping so long as the excitation level exceeds a critical threshold. While frequency shift-based sensing sensitivity is highly influenced by various noise forms and limited by hardware resolution capability, bifurcation-based sensing sensitivity and resolution is theoretically constrained only by thermomechanical noise and Brownian motion [9–11], which is an enticing potential for the mass/force sensing field. For MEMS, bifurcations associated with parametric resonance [12–15], dynamic pull-in [16], and the saddle-node of a softening Duffing oscillator [17] have demonstrated significant mass sensing sensitivity enhancement. Additional benefits include substantially less deterioration of detection performance due to measurement noise than frequency shift-based approaches [12,18,19] and some bifurcation-based approaches offer reduced implementation complexity by eliminating phase-tracking hardware [16,17].

There have been two primary protocols demonstrated for bifurcation-based microscale mass sensing. Both employ the foundational idea that mass accumulation affects bifurcation conditions. In the first, an excitation frequency sweep towards a bifurcation frequency is conducted and the frequency which triggers the jump event is determined; repeating the process yields a time-variation of the shift due to added mass. A bifurcation analysis of Mathieu′s equation for the parametric resonator provides an expression of jumping frequency as a function of system parameters: thus, specific change in the frequency denotes a quantity of adsorbed mass [13]. While the success of this approach is less susceptible to measurement noise and damping than frequency shift-based sensing methods [12,14,18], the technique is still vulnerable to early or delayed jump events [20,21] and requires a collection of active control and tracking hardware. The second protocol is a passive technique that employs a constant frequency excitation just below the bifurcation, allowing for mass adsorption to reduce the critical frequency and induce the jump [16,17]; thus, the second approach detects a threshold. Because mass adsorption may be very slow, the challenges to reliably utilize this technique are tied to the intrinsic phenomena associated with prolonged excitation near bifurcation, including noise-induced transitions [20] and period-doubling cascades [22–24]. Finally, while additional bifurcations due to super- and subharmonics have been demonstrated [16], no bifurcation-based mass sensing works have yet explored means by which to sequentially employ such phenomena to determine the mass adsorbed in the time span between consecutive jumps, thereby achieving mass quantification without excess control and tracking hardware. Therefore, of the remaining challenges in the area of bifurcation-based mass sensing, some key issues are: utilizing bifurcations to passively quantify mass adsorption over time without need for active hardware; avoiding prolonged excitation near the bifurcation frequency which may induce strongly nonlinear phenomena and inhibit reliable detection; and a degree of sensor adaptability to changing testing conditions which may help optimize sweeping through bifurcations for reliable jump event activation and hence mass detection.

To address these concerns, this work presents an alternative bifurcation-based sensing protocol and sensor architecture which conceptually combines frequency shift- and bifurcation-based detection techniques. In the following sections, the operational principle and system architecture are described. Analytical treatment of the system is conducted and validated by experiments to demonstrate the detection strategy and sensor versatility and to initially evaluate noise sensitivities. Further experimental examples verify the successful utilization of the approach and a summary discussion is provided to review the advancements offered by the new bifurcation-based mass sensing system.

2. Architecture and operational principle detail

2.1. Bistable sensor component and capabilities

In contrast to the past bifurcation-based sensor architectures, the sensor in this study utilizes a statically and dynamically bistable element. Bistable MEMS enable broad functionality and as a result have been the focus a wide body of recent research. Numerous studies have explored their application as switches [25,26], valves or actuators [27,28], and non-volatile memory [29,30]. The nonlinearities of bistable MEMS have been probed in detail regarding electrostatic actuation dependence [31,32], imperfections and deviations from ideal fabrication [33–35], and snap through activation [36,37].

A key advantage of bistable elements for bifurcation-based mass sensing is exemplified by evaluating a characteristic steady-state response amplitude to excitation level profile, two examples of which are shown in Fig. 1 as computed from the derivations of the authors′ earlier investigation [38]. For given normalized excitation frequency ω, the level of harmonic excitation h may lead to either small amplitude (intrawell or single-well) or highly energetic (interwell or cross-well) responses. With the bistable element initially oscillating around a stable equilibrium, for example point A in Fig. 1(a), gradually increased excitation level will lead to triggering a bifurcation that induces the energetic interwell response, the B to C jump which represents a significant increase in steady-state response amplitude. Once the bistable element is captured in the interwell response, reduction of excitation level follows a hysteretic trajectory such that the energetic response is sustained down to the critical point E. At this point, further decrease in excitation level activates the E to F bifurcation downwards in amplitude. When excited at a different steady-state excitation frequency, Fig. 1(b), several hysteretic trajectories are observed because three unique dynamics may be induced: a low amplitude intrawell, a higher amplitude intrawell, and energetic interwell responses.