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Exact vibration solutions for cross-ply laminated plates with two opposite edges simply supported using refined theories of variable order

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ABSTRACT

This paper presents exact solutions for free vibration of rectangular cross-ply laminated plates with at least one pair of opposite edges simply supported using refined kinematic theories of variable order. Exact natural frequencies are obtained using an efficient and unified formulation where the solving set of second-order differential equations of motion and related boundary conditions are expressed at layer level in terms of so-called fundamental nuclei having invariant properties with respect to the order of the plate theory. The nuclei are then appropriately expanded according to the number of layers and the order of the theory and the resulting equations are transformed into a first-order model whose solution is obtained by using the state space concept. In this way, the mathematical effort needed to derive analytical solutions is highly reduced. Both higher-order equivalent single-layer and layer-wise theories are considered in this study. Comparisons with other exact solutions are presented and useful benchmark frequency results for symmetric and un-symmetric cross-ply laminates are provided.

1. Introduction

Exact vibration analysis of structural elements like beams, plates and shells can be regarded as the theoretical foundation of almost all approximate solution methods. Exact vibration solutions can be relevant for understanding the dynamic response and performing quick parametric and optimization studies. Furthermore, they can serve as a valuable reference for validating numerical methods on their convergence and accuracy and as a basis for developing advanced modelling techniques such as the dynamic stiffness method and the superposition method [1].

By restricting the analysis to plate problems, mathematically exact solutions are typically available as closed-form solutions and series solutions [2]. It is well known that the most common series solution for plates is the so-called Navier-type solution. In 1820, Navier introduced a simple method for bending analysis of rectangular plates based on the expansion of the displacement field and the load in a double trigonometric series which identically satisfies the boundary conditions of the problem. Exact results can be obtained for specially orthotropic laminates with all edges simply supported [3]. As in the case of bending, the same double Fourier series can be used for vibration and buckling problems.

It is also well known that exact solutions do exist for rectangular specially orthotropic laminates having one pair of opposite edges simply supported and the remaining two edges having any possible combination of free, simple support or clamped conditions [3]. In this case, the displacement is assumed to be expanded in a single trigonometric series along the

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direction normal to the pair of opposite simply supported edges. This form of solution is typically referred to as a Lévy-type solution for both static and dynamic problems. However, as stated by Leissa [4], it was first used by Voigt for transverse vibration analysis in 1893, six years before Lévy proposed the same type of solution for solving the plate bending problem.

Exact transverse and in-plane free vibration analysis of isotropic thin plates with at least two opposite edges supported was first provided by Leissa [4] and Gorman [5], respectively. The remarkable work by Hashemi and Arsanjani [6] on moderately thick plates using the Mindlin theory can be considered as a counterpart of that undertaken much earlier by Leissa for thin plates. Exact vibration solutions of isotropic multi-span and stepped rectangular plates were presented by Xiang and coworkers [7–10]. More recently, Voigt-type solutions for free transverse vibration of thick plates have also been derived via the third-order shear deformation theory [11].

The first-known exact solutions for laminated plates having one pair of opposite simply supported edges are presented in a series of papers by Khdeir, Reddy and Librescu [12–17]. Assuming a single series solution in one direction, the equations of motion are transformed into a set of ordinary differential equations in the other direction. This set is further transformed into a first-order state space model whose general solution is applied to the boundary conditions to obtain the natural frequencies of the problem. Exact eigenfrequencies of symmetric cross-ply and antisymmetric angle-ply laminated plates are generated using the classical lamination plate theory (CLPT), the first-order shear deformation theory (FSDT) and the third-order shear deformation theory (TSDT) originally proposed by Vlasov for isotropic structures and then extended by Reddy to composite plates and shells [18]. At a later stage, the same method has been applied to plates modelled according to a second-order shear deformation theory [19] and a two-variable refined theory [20].

All the above-cited analytical works are based on two-dimensional (2-D) plate theories essentially built according to a Newtonian approach, where the kinematic variables of the displacement model represent physical quantities like translations, rotations and warping. They neglect plate thickness stretching and are simple enough to yield economical models that could be handled rather easily by analytical techniques. However, they may introduce overly simplified assumptions concerning the three-dimensional (3-D) kinematics of deformation of the plate. Indeed, multilayered constructions are typically characterized by high transverse shear and normal deformation and by a displacement field with discontinuous derivatives along the thickness direction (so-called *zig–zag* effect). Such complicating effects are completely discarded or only approximately captured by FSDT and TSDT. The accuracy of Newtonian-based plate theories in predicting the laminate vibration behavior is even worse when the thickness-to-length ratio of the plate increases and the frequency range of interest widens.

Owing to the complex nature of the 3-D deformation of laminated plates, many refinements of FSDT have been proposed in the literature to improve the accuracy of 2-D plate models without resorting to a cumbersome fully 3-D analysis. They are typically referred to as refined or higher-order shear deformation theories [21–26] and belong to the class of theories developed according to a Lagrangian approach, where each kinematic variable of the assumed displacement models can be considered as a generalized coordinate without a direct physical meaning. Generally speaking, Lagrangian-based plate theories can be classified as equivalent single-layer (ESL) models, where the classical FSDT displacement form is enriched with higher-order terms as series expansion of the thickness coordinate, and layer-wise (LW) models, in which a different displacement field is postulated in each layer and appropriate continuity conditions are enforced at each layer interface. The number of expansion terms for each displacement variable included into the plate model is referred to as the order of the theory.

The disadvantage of refined plate theories is the complexity of the resulting models, which are lengthy and tedious to derive and difficult to solve by analytical methods. To the author's best knowledge, exact Voigt-type solutions based on a refined theory have been obtained only recently by Boscolo [27]. In this work, free vibration of rectangular laminated plates is solved using a first-order layer-wise theory. Exact eigenfrequencies are validated by comparison with available analytical 3-D and 2-D Navier-type solutions. The approach developed in Ref. [27] is used by Boscolo and Banerjee [28] within the framework of the dynamic stiffness method. Although the first-order layer-wise displacement model may suffer from some limitations in terms of accuracy and efficacy, the effort is remarkable.

The aim of the present work is to present an efficient, unified and somehow automatic method to provide exact vibration solutions of thin and thick cross-ply laminates with at least two opposite edges simply supported. It can be considered as a generalization of what presented in Ref. [27]. The novel procedure introduced here overcomes the shortcomings of the previous formulations which were limited to plate models derived from a single theory with fixed kinematics (i.e., fixed order). Using the present approach, the solving equations must not be re-derived when a different order of the theory is adopted and thus the mathematical effort needed to obtain analytical solutions is substantially reduced. In particular, both two-dimensional ESL and LW theories of variable order are considered. As a result, a considerable number of new exact frequency results are obtained which can be useful as benchmark solutions for future comparison.

2. Governing equations at layer level

2.1. Preliminaries

Consider an unloaded cross-ply laminated rectangular plate of length a, width b and thickness h (see Fig. 1). The plate consists of N_ℓ layers, which are assumed to be homogeneous and made of orthotropic material of mass density ρ^k. The kth