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A chiral elastic metamaterial beam for broadband vibration suppression

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ABSTRACT

One of the significant engineering applications of the elastic metamaterial (EMM) is for low-frequency vibration attenuation because of its unusual low-frequency bandgap behavior. However, the forbidden gap from many existing EMMs is usually of narrow bandwidth which limits their practical engineering applications. In this paper, a chiral-lattice-based EMM beam with multiple embedded local resonators is suggested to achieve broadband vibration suppression without sacrificing its load-bearing capacity. First, a theoretical beam modeling is suggested to investigate bandgap behavior of an EMM beam with multiple resonators. New passbands due to dynamic interaction between resonators are unpleasantly formed, which become a design barrier for completely broadband vibration suppression. Through vibration attenuation factor analysis of the resonator, an EMM beam with section-distributed resonators is proposed to enable broadband vibration attenuation function. Required unit number of the resonator in each section is quantitatively determined for complete vibration attenuation in a specific frequency range. Finally, the chiral-lattice-based EMM beam is fabricated, and experimental testing of the proposed structure is conducted to validate the design.

1. Introduction

EMMs have gained much attention due to their unique microstructure designs to achieve effective dynamic material properties which cannot be observed in nature [1]. The working principle of the EMM is to use man-made microstructures (local resonators) on a scale much less than its working wavelength. Therefore, low-frequency bandgap can be observed in the EMM with small dimensions, within which the wave/vibration energy cannot propagate. The unusual low-frequency bandgap in such composite was explained by the negative effective mass density through equivalent discrete mass-spring systems [2–5].

One of the significant engineering applications of the EMM is to achieve the low-frequency vibration attenuation. Different from the Bragg scattering mechanism in phononic crystals [6,7], the locally resonant (LR) mechanism could be easily tuned through proper microstructure design, and low-frequency vibration energy could be quickly attenuated within only a small amount of the periodic microstructures [8]. Therefore, no gigantic meta-structure is needed to shield the structural subject from the low-frequency vibration or wave loading. Engineering structures such as rods, beams and plates

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with desired LR microstructure designs were implemented for vibration suppression. Xiao et al. [9] investigated wave propagation and vibration transmission in elastic rods containing periodically attached multi-degree-of-freedom spring-mass resonators. Yu et al. [10] studied the flexural wave propagation in a beam with many spring-mass subsystems as bending wave absorbers. Chen et al. [11] analytically and experimentally studied the behavior of bending wave propagation in a sandwich beam with internal resonators. However, the forbidden gap from the current EMM design is usually of narrow bandwidth, which significantly limits its potential engineering applications. To address this problem, bandgaps in acoustic metamaterials with multi-resonators were investigated [12]. It was found that the bandgaps can be tuned by varying physical parameters of internal resonators. Pai [13] theoretically demonstrated that the longitudinal broadband wave absorption can be achieved in a bar structure with distributed absorbers related to different bandgap frequency ranges in different sections. Based on the study, a metamaterial beam was also suggested to achieve broadband vibration absorption [14]. Similarly, flexural wave propagation in the beam consisting of multiple damped spring-mass resonators with slightly different resonant frequencies was also investigated by Xiao et al. [15]. Broader bandgaps were found at frequencies both below and around the Bragg condition. A chiral-lattice-based metacomposite beam was recently proposed by integrating periodic chiral lattice with LR inclusions for low-frequency vibration attenuation applications [16]. The vibration attenuation function was demonstrated through the numerical analysis of the band diagram. The major advantage of the proposed beam is that the significant vibration attenuation is localized within the structure, which requires no additional structural components. Additionally, the chiral structure beam can still be made from stiff and high strength materials so as not to sacrifice the load-bearing capacity. To accomplish the chiral-lattice-based EMM for vibration attenuation in a broad frequency regime, the EMM beam with multiple inner resonators should be properly designed and the experimental validation of the design should be conducted.

In this paper, a chiral-lattice-based EMM beam with multiple local resonators is numerically and experimentally studied for the broadband vibration suppression by utilizing their individual bandgaps. First, based on the Timoshenko beam theory (TBT) and transfer matrix method (TMM), theoretical modeling of an EMM beam with multiple local resonators is performed for vibration analysis. The undesirable new passbands are observed due to dynamic interaction between the different resonators, which become a major design barrier to form complete vibration attenuation in a desired frequency regime. Through vibration attenuation factor analysis, a section-distributed design of multiple local resonators is suggested to achieve completely broadband vibration suppression and required unit number of the resonator in each section is quantitatively determined. Finally, the chiral-lattice-based EMM beam is fabricated, and experimental frequency response testing is conducted to validate the proposed design as well as the theoretical modeling.

2. Bending vibration in a beam with multiple local resonators

The vibration band structure of a beam with a single LR structure has been investigated based on the transfer matrix theory [17]. In the study, to form a broad forbidden band, we implement this method to obtain the band structure of the EMM beam with multiple local resonators. Attention will be paid on the understanding of dynamic interaction among different local resonators and its effects on vibration transmission. To clearly illustrate the problem, a simple model of the beam with multiple LR units is studied as shown in Fig. 1a. Each unit consists of s subsystems in which mass-spring resonators are attached to the beam at a spacing of a along x direction. Each subsystem consists of two parts, beam segment and local resonator, which is represented by an elastic spring k and a mass m_j, $j=1, 2, 3, \ldots s$, as shown in Fig. 1b. The lattice constant of the periodic system is denoted as $b=sa$. The x axis of the coordinate system is along the central line of the beam.

The governing equation of the free bending vibration of a Timoshenko beam can be written as follows:

$$\frac{EI}{\rho\overline{A}}\frac{\partial^4 v(x,t)}{\partial x^4}-\left(\frac{\rho I}{\rho\overline{A}}+\frac{EI}{\kappa G\overline{A}}\right)\frac{\partial^4 v(x,t)}{\partial x^2\partial t^2}+\frac{\partial^2 v(x,t)}{\partial t^2}+\frac{\rho I}{\kappa G\overline{A}}\frac{\partial^4 v(x,t)}{\partial t^4}=0, \tag{1}$$

where ρ, E, and G are the density, Young's modulus, and shear modulus, respectively; $\overline{A}$ is the cross-section area; κ is the Timoshenko shear coefficient; I is the cross-section-area moment of inertia about the axis perpendicular to x and y axes. Unlike Euler–Bernoulli beam theory which neglects shear deformation, Timoshenko beam with rotary inertia considers the deformation of the beam cross-section, therefore it is more suitable for short beams i.e., those with relatively high cross-sections compared with their lengths, especially when they are subjected to significant shear forces. Since only the steady-state response will be considered in this section, the time factor $e^{i\omega t}$, which applies to all the field variables, will be suppressed. Therefore, the amplitude $Y(x)$ of the bending displacement $v(x,t)$ can be determined as [18,19]

$$Y(x)=Ak_1^{-3}e^{q_1x}+Bk_2^{-3}e^{q_2x}+Ck_3^{-3}e^{q_3x}+Dk_4^{-3}e^{q_4x}, \tag{2}$$

where

$$q_r=(-1)^{[r/2]}\sqrt{[\overline{\alpha}+(-1)^r\sqrt{\overline{\alpha}^2+4\overline{\beta}}]/2},\quad r=1,\ldots,4,\ \overline{\alpha}=-\frac{\rho\omega^2}{E}-\frac{\rho\omega^2}{\kappa G}\ \text{and}\ \overline{\beta}=\frac{\rho\overline{A}\omega^2}{EI}-\frac{\rho^2\omega^4}{E\kappa G},\ \left[\frac{r}{2}\right]$$

is the largest integer less than $r/2$. In Eq. (2), q_r ($r=1, 2, 3, 4$) represent the wavenumbers of the two lowest vibration modes along two directions (positive and negative x directions).