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Frequency modelling and solution of fluid–structure interaction in complex pipelines

Yuanzhi Xu [a,b], D. Nigel Johnston [b], Zongxia Jiao [a,*], Andrew R. Plummer [b]

[a] School of Automation Science and Electrical Engineering, Beihang University, No. 37, Xueyuan Road, Haidian District, Beijing 100191, China
[b] Department of Mechanical Engineering, University of Bath, Bath, BA2 7AY, UK

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ABSTRACT

Complex pipelines may have various structural supports and boundary conditions, as well as branches. To analyse the vibrational characteristics of piping systems, frequency modelling and solution methods considering complex constraints are developed here. A fourteen-equation model and Transfer Matrix Method (TMM) are employed to describe Fluid–Structure Interaction (FSI) in liquid-filled pipes. A general solution for the multi-branch pipe is proposed in this paper, offering a methodology to predict frequency responses of the complex piping system. Some branched pipe systems are built for the purpose of validation, indicating good agreement with calculated results.

1. Introduction

Fluid–Structure Interaction (FSI) describes an explicit coupling between moving fluid and deformable structure, in which fluid acts on the structure with fluidic force whilst simultaneously the fluid is acted upon by movement of the structural boundary. The FSI in a fluid-conveying pipeline can be induced by sudden opening or closing of a valve, sudden start-up or shutdown of a pump, fluid flow ripples and mechanical excitation. This phenomenon has been found in a wide range of fields, ranging from hydraulic and pneumatic fluid power systems, water supply systems, power production, petrochemical industry, and even biological vessels. This paper is focused on the FSI in liquid-filled pipes, especially applied to hydraulic piping systems of complex supports and spatial configurations.

Structural supports will affect the behaviour of a piping system significantly, changing the system's natural frequencies. In this paper, various boundary conditions and middle constraints are studied and included in the pipe system model. Although models of straight, curved and T-shaped pipes have been studied previously, a more complicated system has not yet been intensively researched. In the present work, a general solution of the multi-branch pipe system is proposed, and experiments are carried out.

1.1. Literature of analytical model

Water hammer theory was developed in the 19th century, mostly based on the research of Joukowsky [1], who presented the formula to predict the pressure change ΔP with the velocity change ΔV,

$$\Delta P = \rho_f c_f \Delta V \tag{1}$$

* Corresponding author. Tel.: +86 10 82338938; fax: +86 10 82338910.
E-mail addresses: zxjiao@buaa.edu.cn, zxjiao@vip.sina.com (Z. Jiao).

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Nomenclature

Uppercase letters

A	cross-sectional area
E	Young's modulus
F	external force of constraints
G	shear modulus
I	flexure moment of inertia
J	polar moment of inertia
K	fluid bulk modulus
L	length of pipe
P	fluid pressure
T	external moment of constraints
V	fluid velocity
Y	angular impedance of constraints
Z	linear impedance of constraints

Lowercase letters

c	wave speed
e	thickness of pipe wall
f	force in cross-section
m	pipe moment or extra mass of constraints
r	radii of pipe cross-section
u	pipe displacement
z	distance along the pipe
ρ	density
ν	Poisson's ratio
ψ	pipe rotation displacement
φ	angle between two adjacent coordinate systems

Subscripts

e	external excitation
f	fluid
p	pipe
i	inner
o	outer
x,y	lateral coordinates
z	axial coordinate

Superscripts

$\sim$	Laplace transformed
T	transposed

Matrices and vectors

$\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$	coefficient matrix of FSI model
$\mathbf{D}$	boundary matrix
$\mathbf{I}$	identity matrix
$\mathbf{M}$	field transfer matrix
$\mathbf{N}$	constraint matrix
$\mathbf{Q}$	excitation vector
$\mathbf{R}$	rotation matrix
$\mathbf{T}$	point transfer matrix of T-junction
$\boldsymbol{\phi}$	state vector of 14 variables
$\mathbf{0}$	zero vector/matrix

Although Joukowsky used the sound velocity which takes into account both the compressibility of the fluid and the elasticity of the pipe walls [2], the expression was a one-way coupling excluding structural vibration. Wylie and Streeter [3] and Cai [4] presented the impedance method, and the pipe elasticity was incorporated. It was simple and effective in predicting behaviours of fluid transients, but still not in a coupled way.

The two-way coupling mechanism between the fluid transient and the movement of pipe wall has been defined as three kinds of coupling [2,5,6]. *Poisson coupling* is due to the Poisson effect, in which an oscillatory pressure force results in radial pipe wall dilation and hence axial strain and movement. *Junction coupling* takes place at changed boundaries, such as elbows, valves, junctions and pipe ends due to the unbalanced pressure force acting on an area. *Friction coupling* is due to shear stresses on pipe walls, and is generally considered less significant than the other two couplings.

Basic water hammer equations (two-equation model [2]) could be derived from the Navier–Stokes equation and the continuity equation (Zielke [7]),

$$\frac{\partial V}{\partial t}+\frac{1}{\rho_f}\frac{\partial P}{\partial z}=0, \tag{2}$$

$$\frac{\partial V}{\partial z}+\frac{1}{K}\frac{\partial P}{\partial t}=0. \tag{3}$$

Skalak [8,9] later proposed four linear first-order partial differential equations (PDEs) for the two-way interaction, which was also known as the four-equation model. Wiggert et al. [10] presented an axial four-equation model containing the Poisson coupling, based on the work of Walker and Phillips [11]. The axial four-equation model was then widely used and achieved good predictions of straight pipes [12–14] Zhang et al. [6] utilised the four-equation model to simulate the vibration of a liquid-filled straight pipe in the frequency domain. Eight equations for a curved pipe had been obtained by Davidson and Smith [15], extended by Valentin et al. [16] and Hu and Phillips [17], where Poisson and junction coupling were taken into account.

A fourteen element vector was first used by Davidson and Samsury [18] and applied on the simulation of a pipeline consisting of straight and curved pipes. Wilkinson [19] presented the 14-equation model where equations of motion were