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A study on waviness induced vibration of ball bearings based on signal coherence theory

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ABSTRACT

This paper focuses on the effects of waviness on vibration of ball bearings. An experimental analysis method is developed by adopting signal coherence theory of multiple-inputs/single-output (MISO) system. The inputs are waviness excitations of the inner and outer races, and the output is vibration response of the outer ring. Waviness excitation signals are first derived from the manufacturing deviations, and found to be strongly coherent in low frequency range. Virtual input signals are then introduced by the method of orthogonalization. In both cases of vibration acceleration and speed responses, the cumulated virtual input–output coherence function verifies that the first peak region of vibration spectrum is mainly induced by the waviness excitations. In order to distinguish the contributions of the inner and outer races, coherence functions of the virtual inputs with real inputs are calculated, and the results indicate that the outer race waviness contributes more to vibration than the inner race waviness does in the example. Further, a multi-body dynamic model is constructed and employed to frequency response analyses. It is discovered that the waviness induced spectral peak frequency is close to the natural frequency of bearing.

1. Introduction

Ball bearings are widely used in high speed machine tool spindles. For this kind of machine tool spindle, ball bearings act as not only the media but also the sources of vibration. Assuming that there are no assembly errors and the lubricant has no impurities, the causes of ball bearings induced vibration generally involve:

(i) flexural deformation of outer ring,
(ii) variable elastic compliance,
(iii) geometry imperfections.

Flexural deformation of outer ring occurs due to the uniformly distributed contact loads on outer ring when the bearing is subjected to an axial external load, giving rise to a vibration frequency which is equal to ball passage frequency, that is, the cage rotational frequency multiplied by the number of balls [1]. The elastic compliance of ball bearings is time-variable because the angular position of balls, with respect to the radial load, continually changes with time, resulting in that the

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Nomenclature

A	Amplitude of the first harmonic
$C_{x_1x_2}$	Coherence function of the waviness excitation signals of outer race and inner race
f_j^{ic}	Excitation forces between the jth ball and the inner ring
f_j^{oc}	Excitation forces between the jth ball and the outer ring
f_{or}^z	Axial force preloaded on the outer ring
f_{s1}	Equivalent sampling frequency of the outer race waviness excitation signal
f_{s2}	Equivalent sampling frequency of the inner race waviness excitation signal
$G_{x_1x_1}$	PSD of the outer race waviness excitation signal
$G_{x_2x_2}$	PSD of the inner race waviness excitation signal
$G_{x_1x_2}$	CSD of two waviness excitation signals
I_{or}^x	Moment of inertia of the outer ring around the x-axis
I_{or}^y	Moment of inertia of the outer ring around the y-axis
I_{ir}^z	Moment of inertia of the inner ring around the z-axis
J_b	Moment of inertia of a ball
m_b	Mass of a ball
n	Order of harmonics
N	The maximum order of harmonics
R_b	Radius of a ball
R_{ir}^g	Curvature radius of the inner ring groove
R_{or}^g	Curvature radius of the outer ring groove
R_p	Radius of the bearing's pitch circle
s	Amplitude decay coefficient for the subsequent wavenumbers
S_j	Nonlinear terms in Eqs. (14) and (15), $j=1, 2, \ldots, 3Z+5$
T_{ir}	Kinetic energy of the inner ring
T_{or}	Kinetic energy of the outer ring
T_b	Total kinetic energy of the balls
$u_{or}^x, u_{or}^y, u_{or}^z$	Vibration displacements of the outer ring in the x, y and z-directions
$u_{bj}^x, u_{bj}^y, u_{bj}^z$	Vibration displacements of the jth ball in the x, y and z-directions
V_j	Sum of the potential energy in the jth inner and outer contacts
VC_{yx1}	Coherence function of the virtual input signal $\tilde{x}_1$ with the output signal
VC_{yx2}	Coherence function of the virtual input signal $\tilde{x}_2$ with the output signal
W_j^{ir}, W_j^{or}	Waviness of the inner and outer races
x,y,z	Cartesian fixed global frame of reference
Z	Number of the balls
ω_b	Rotational speed of the balls relative to the cage
ω_c	Revolution speed of the balls
ω_{ir}	Rotational speed of the inner ring
ω_{ni}	Undamped natural frequencies
ϕ_{or}^x	Vibration angular displacement of the outer ring around the x-axis
ψ_{or}^y	Vibration angular displacement of the outer ring around the y-axis
φ_j	Tangential coordinate of the jth ball
$\delta_j^{ic}, \delta_j^{oc}$	The variable elastic deformations in the jth inner and outer contacts
$\delta_{init}^{ic}, \delta_{init}^{oc}$	The stable elastic deformations in the jth inner and outer contacts
γ_n	A number averagely distributed between 0 and 2π
$\kappa_j^{ic}, \kappa_j^{oc}$	Coefficient of contact stiffness
$\mathbf{f}$	Generalized force vector
$\mathbf{G}$	Cross-spectral matrix of the waviness excitation signals
$\mathbf{K}$	stiffness matrix
$\mathbf{K}_d$	Diagonalized stiffness matrix
$\mathbf{M}$	Mass matrix
$\mathbf{s}_i$	modal vectors
$\bar{\mathbf{s}}_i$	Normalized modal vectors
$\bar{\mathbf{S}}$	Modal matrix
$\mathbf{u}$	Generalized coordinate vector
$\mathbf{v}$	Modal coordinate vector

vibration frequency also equals ball passage frequency [2,3]. Geometry imperfections can be classified into two categories, one of which is the local defect in balls and inner and outer races such as cracks, pits and spalls which surely generate vibration in the ball-race's contacts [4–7], the other one is distributed defects such as waviness of the rolling contact surfaces, which can give rise to fluctuations continuously in the dynamic reaction of rolling contacts. Because of the irregularity of surface topography and interaction between two or more surfaces, the waviness induced vibration is much more complicated in practice.

In 1965, Tallian and Gustafsson [8] summarized the vibration frequencies generated by multi sets of special orders of waviness, but the results are valid only for the particular kind of mounting of the vibration tester used in the bearing industry. Yhland [9] presented a detailed analysis of waviness measurement and made a comparison between the theoretical and measured spectra of vibration. Meyer et al. [10] found that the lack of roundness of races generates sidebands around a series of “normal” tones, and the frequency difference is proportional to the number of wave. Rahnejat and Gohar [11] performed a numerical solution for the vibration response of a rotating rigid shaft subjected to the inner race waviness of bearings, obtaining that a response frequency equaling the shaft speed times the number of waves is significant in addition to the ball passage frequency. Wardle [12,13] and Lynagh et al. [14] developed more general analytical expressions of the vibration forces and moments in bearings, obtaining the principal frequencies resulted from the waviness