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Structural damage measure index based on non-probabilistic reliability model

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ABSTRACT

Uncertainties in the structural model and measurement data affect structural condition assessment in practice. As the probabilistic information of these uncertainties lacks, the non-probabilistic interval analysis framework is developed to quantify the interval of the structural element stiffness parameters. According to the interval intersection of the element stiffness parameters in the undamaged and damaged states, the possibility of damage existence is defined based on the reliability theory. A damage measure index is then proposed as the product of the nominal stiffness reduction and the defined possibility of damage existence. This new index simultaneously reflects the damage severity and possibility of damage at each structural component. Numerical and experimental examples are presented to illustrate the validity and applicability of the method. The results show that the proposed method can improve the accuracy of damage diagnosis compared with the deterministic damage identification method.

1. Introduction

Within their service lives, civil structures are inevitably subjected to deterioration and damage resulting from environmental erosion, overloading, fatigue, material aging, or other unexpected factors. Damage detection at the possible earliest stage pervades in the civil, mechanical, and aerospace engineering communities [1]. Because of the limitations in experimental methods, where the vicinity of the damage must be known a priori and the portion of the structure being inspected must be readily accessible, vibration-based damage detection methods have been developed extensively since the 1990s [2].

The dynamic properties of the frequency domain (such as natural frequency, mode shape, mode shape curvature, modal flexibility, and modal strain energy) [3–6] or the responses in the time domain [7–9] have been adopted as indicators of damage. In practice, measurement data are always limited and contain noises or errors to some extent. To reduce the effects of the uncertainty of limited measurement data on the damage diagnosis, researchers are searching for indicators with high sensitivity to damage so that the useful information is not drowned by the noises [10]. On the other hand, statistical damage identification methods have been proposed to address various uncertainties involved [11].

Collins et al. [12] first derived a statistical identification procedure by treating the initial structural parameters as normally distributed random variables with zero means and specific covariance. Xia and Hao [13] developed a statistical

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damage identification algorithm accounting for the effects of measurement noise in the natural frequencies and variations in the finite element (FE) model, and derived the probability of damage existence. They further extended the statistical approach to the case with combined frequency and mode shape data for structural damage identification [14]. Based on acceleration responses, Li and Law [15] analyzed the influence of the uncertainty of system parameters and the measurement data on damage identification. Yeo et al. [16] presented a damage assessment algorithm for framed structures using static responses with a regularization technique, in which statistical distributions of the system parameters with a set of noise-polluted measurement data were derived by the perturbation method and then the damage was assessed by a statistical hypothesis test approach. To avoid damage identification induced by the measurement noise, a probabilistic method was proposed to identify the structural damages with uncertainties under unknown input [17].

In these methods, the statistical distributions of the uncertainties are assumed to be known (usually as Gaussian distribution). In practice, however, the uncertainty sources are complicated, and experimental data under a particular condition are insufficient. The probabilistic distributions of the uncertainties are usually not available. In this regard, the non-probabilistic interval analysis has been developed [18,19] for damage identification, in which the uncertainty bounds, rather than the probabilistic distributions, of the measurement data are employed. Wang et al. [20,21] applied the interval analysis technique for structural damage identification using the bounded natural frequencies and the static displacements of the structures, respectively. Damage identifications for a steel cantilever beam and a steel cantilever plate were performed by the proposed non-probabilistic method in comparison with the probabilistic approach [20].

In both probabilistic and non-probabilistic approaches, the nominal (or mean value of) stiffness reduction of each element and the probability (or possibility) of damage are separately provided to assess the damage of the structures. However, a significant stiffness reduction may have a low probability of damage because probability is associated with both the mean value and the variance. For the same reason, a small stiffness reduction may have a relatively high probability. Therefore, using the mean value of the stiffness reduction or the probability of damage alone may not come up with an accurate damage assessment.

In this paper, the stiffness reduction and possibility of damage are combined as a new damage measure index (DMI). The non-probabilistic interval analysis framework is adopted to identify the stiffness parameter interval from the measured uncertain frequencies and mode shapes. The possibility of damage of each structural member is calculated by virtue of the non-probabilistic, set-theoretic reliability theory [22] from the member stiffness intervals in the undamaged and damaged states. The DMI is defined as the product of the nominal stiffness reduction and possibility of damage. It simultaneously reflects the degree and possibility of damage for each structural component. A numerical example of a 15-bar truss structure and an experimental example of a one-span steel portal frame are presented to demonstrate the effectiveness of the proposed method.

2. Deterministic FE model updating using both frequencies and mode shapes

The free vibration problem of an undamped structure with N degrees of freedom can be expressed as

$$\mathbf{K}\boldsymbol{\phi}_i = \lambda_i \mathbf{M}\boldsymbol{\phi}_i, \; i = 1, 2, \ldots, N \tag{1}$$

where $\mathbf{M}$ is the $N \times N$ mass matrix, $\mathbf{K}$ is the $N \times N$ stiffness matrix, $\mathbf{x}(t)$ and $\ddot{\mathbf{x}}(t)$ are the displacement and acceleration vectors, respectively, and λ_i and $\boldsymbol{\phi}_i$ are the ith eigenvalue and mass-normalized mode shape, respectively. If changes occur in the structural parameters, the eigenvalue problem is expressed as

$$\mathbf{K}_c\boldsymbol{\phi}_{ci} = \lambda_{ci} \mathbf{M}_c\boldsymbol{\phi}_{ci}, \; i = 1, 2, \ldots, N \tag{2}$$

where $\mathbf{K}_c$, $\mathbf{M}_c$, λ_{ci}, and $\boldsymbol{\phi}_{ci}$ are the corresponding quantities in the changed state.

For the FE model of the structure, $\mathbf{K}$ can be expressed in the following non-negative parameter decomposition form:

$$\mathbf{K} = \sum_{i=1}^{m} \alpha_i \mathbf{K}_i = \alpha_1 \mathbf{K}_1 + \alpha_2 \mathbf{K}_2 + \ldots + \alpha_m \mathbf{K}_m \tag{3}$$

where m is the number of elements in the structure, α_i is the initial elemental stiffness parameter (ESP), and $\mathbf{K}_i$ is the ith element stiffness matrix divided by α_i. Similarly, $\mathbf{K}_c$ is set up as

$$\mathbf{K}_c = \sum_{i=1}^{m} \alpha_{ci} \mathbf{K}_i = \alpha_{c1} \mathbf{K}_1 + \alpha_{c2} \mathbf{K}_2 + \ldots + \alpha_{cm} \mathbf{K}_m \tag{4}$$

The model updating is based on the relationship between the measured vibration characteristics and the ESP using the first-order Taylor series expansion as [12]

$$\begin{pmatrix} \boldsymbol{\lambda}_c \\ \boldsymbol{\phi}_c \end{pmatrix} = \begin{pmatrix} \boldsymbol{\lambda} \\ \boldsymbol{\phi} \end{pmatrix} + \mathbf{S}(\boldsymbol{\alpha}_c - \boldsymbol{\alpha}) \tag{5}$$

where $\mathbf{S}$ is the sensitivity matrix of the modal properties with respect to the ESPs [23].