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Free vibration analysis of a rotating hub–functionally graded material beam system with the dynamic stiffening effect

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ABSTRACT

A comprehensive dynamic model of a rotating hub–functionally graded material (FGM) beam system is developed based on a rigid–flexible coupled dynamics theory to study its free vibration characteristics. The rigid–flexible coupled dynamic equations of the system are derived using the method of assumed modes and Lagrange's equations of the second kind. The dynamic stiffening effect of the rotating hub–FGM beam system is captured by a second-order coupling term that represents longitudinal shrinking of the beam caused by the transverse displacement. The natural frequencies and mode shapes of the system with the chordwise bending and stretching (B–S) coupling effect are calculated and compared with those with the coupling effect neglected. When the B–S coupling effect is included, interesting frequency veering and mode shift phenomena are observed. A two-mode model is introduced to accurately predict the most obvious frequency veering behavior between two adjacent modes associated with a chordwise bending and a stretching mode. The critical veering angular velocities of the FGM beam that are analytically determined from the two-mode model are in excellent agreement with those from the comprehensive dynamic model. The effects of material inhomogeneity and graded properties of FGM beams on their dynamic characteristics are investigated. The comprehensive dynamic model developed here can be used in graded material design of FGM beams for achieving specified dynamic characteristics.

1. Introduction

With the development of modern aerospace technology, traditional isotropic materials are not able to meet the needs of practical applications; a variety of new types of composite materials have emerged. Functionally graded materials (FGMs) are one kind of advanced materials that have attracted widespread attention of researchers due to its unique superiority of heat resistance, high strength, and light weight [1]. By intelligently designing material constitutions and gradient distributions of FGMs, structures made of them can have excellent dynamic and thermodynamic characteristics. In recent years, FGMs are showing better prospects than traditional laminated composites [2].

There is much research dealing with general mechanics of structures made of FGMs [3]. The linear and nonlinear dynamics of FGM beams were studied in Refs. [4–11]. Over the last decade, the dynamic modeling and analysis of rotating blades made of FGMs has become a topic of considerable research. To the best of the authors' knowledge, the works by Oh

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et al. [12,13] are the first ones dealing with rotating beams made of FGMs. On the basis of the study in Ref. [13], Fazelzadeh and Hosseini [14] and Fazelzadeh et al. [15] investigated the dynamic characteristics of a rotating thin-walled blade made of FGMs in high temperature, supersonic gas flow by using Galerkin method and the differential quadrature method. The effects of the Mach number, rotating speed, geometric parameters, and material properties on the natural frequencies of the blade were examined. The convergence of the two methods was compared and excellent agreements were observed. However, the dynamic stiffening effect was not taken into account in Refs. [12–15] as their interest was in the thermo-elastic effect related to the graded properties. Piovan and Sampaio [16] introduced a new finite element model to study the dynamic behavior of rotating beams made of FGMs. It accounted for shear deformation and the nonlinear strain–displacement relationship, making it possible to model the dynamic stiffening effect. Recently, Attarnejad and Shahba [17,18] developed basic displacement functions for deriving shape functions in the finite element method for free vibration analysis of non-prismatic beams. The methodology has been used to solve rotating, axially functionally graded tapered beams [19]. Rajasekaran et al. [20] studied the free vibration problem of a rotating, axially functionally graded tapered Timoshenko beam by using the differential transformation method and the differential quadrature element of the lowest order and obtained some valuable results.

Systems with a large rigid body motion and small elastic vibration are referred to as rigid–flexible coupled dynamic systems. Structures such as helicopter rotor blades, robotic manipulators, turbine blades, and spinning space structures can be modeled as rotating hub–beam systems to study their dynamic characteristics. Since Kane et al. [21] revealed the dynamic stiffening phenomenon, many researchers have developed new methodologies to capture the dynamic stiffening terms in the dynamic equations [22–26]. With the consideration of a second-order coupling term between longitudinal and transverse vibrations, Cai et al. [27] introduced a first-order approximate coupled (FOAC) model of a rotating hub–beam system that is applicable to both small and large angular velocities of the hub. Interesting frequency veering and mode shift phenomena [28] occur in a rotating hub–beam system when the dynamic stiffening effect is included [29]. Such kinds of coupled vibration behaviors have received considerable attention [30,31]. As there is little work that discusses coupled vibration behaviors of rotating FGM beams from small to large angular velocities, the frequency veering and mode shift behaviors will be studied from a rigid–flexible coupled dynamics viewpoint in this work.

The rigid–flexible coupled dynamic equations of a rotating hub–FGM beam system are derived in Section 2 using the method of assumed modes and Lagrange's equations of the second kind. The natural frequencies of a rotating hub–homogeneous beam system are first calculated in Section 3.1 using the FOAC model without considering the chordwise bending and stretching (B–S) coupling effect, and compared with those from the traditional hybrid coordinate method [32], also referred to as the zeroth-order approximate coupled (ZOAC) model in Ref. [27]. Both the ZOAC and FOAC models are obtained as a special case from the comprehensive dynamic model developed in Section 2, and the FOAC model is used in the subsequent study of the rotating hub–FGM beam system due to its advantage over the ZOAC model for relatively large angular velocities. The natural frequencies and mode shapes of a rotating FGM beam with the B–S coupling effect included are calculated in Section 3.2 and compared with those with the B–S coupling effect neglected. A two-mode model is developed in Section 3.3 to predict the most obvious frequency veering behavior observed in Section 3.2 between two adjacent modes associated with a chordwise bending and a stretching mode. The angular velocities of the rotating FGM beam with which the frequency veering behavior occurs, referred to as the critical veering angular velocities of the beam, are analytically determined from the two-mode model for the most obvious frequency veering behavior. The effects of material constitution and gradient distribution of the chordwise bending natural frequencies of rotating FGM beams are discussed in Section 4.

2. Physical model and basic equations

2.1. Description of the problem

As shown in Fig. 1, a uniform flexible beam made of FGMs is attached to a rigid hub rotating about the vertical Z_0 axis in a fixed coordinate system $O_0X_0Y_0Z_0$. The radius of the hub is R and τ is an external torque acting on the hub. The rotary inertia of the hub is J_{oh}, and L, b, and h are the length, width, and thickness of the beam, respectively.

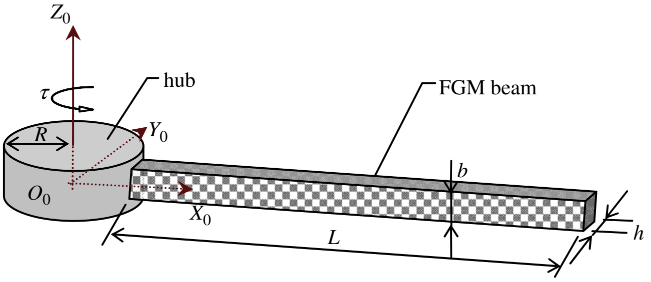

Fig. 1. Schematic of a rotating hub–FGM beam system.