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An approach to aerodynamic sound prediction based on incompressible-flow pressure

P. Martínez-Lera [a,*], C. Schram [b], H. Bériot [a], R. Hallez [a]

[a] LMS International, 3001 Leuven, Belgium
[b] von Karman Institute for Fluid Dynamics, 1640 Rhode-St-Genèse, Belgium

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ABSTRACT

Curle's analogy provides a solution to Lighthill's equation to predict flow-generated sound in the presence of rigid boundaries. Nevertheless, Curle's solution requires the flow pressure, including its acoustic component, to be known in the source region. If the pressure corresponds to an incompressible-flow description instead and the surface is not acoustically compact, significant errors can arise in the acoustic prediction. In this work, it is argued that flow wall pressure can be used to define appropriate boundary conditions of an equivalent acoustic boundary value problem for an arbitrary geometry, and a formulation of a boundary condition based on incompressible-flow pressure is proposed. The theoretical analysis suggests that if the flow is incompressible, the error has the leading order of a dipole plus a quadrupole for Curle's analogy and of just a quadrupole for the proposed alternative approach, thus making the latter more accurate when dipole sources are dominant. A numerical test case is presented as a proof of concept, consisting of a trailing edge noise problem due to the flow past a slender body.

1. Introduction

The acoustic analogy proposed by Lighthill [1] remains a widespread approach to compute flow generated sound for low Mach-number flows [2,3]. In the presence of solid rigid boundaries in the region of sound production, the analogy proposed by Curle [4] provides a solution to Lighthill's equation in which the effect of solid boundaries is characterized as dipole sources. When the solid boundary is acoustically compact, Curle's dipoles are the dominant source of sound for low Mach numbers. In other words, the surface increases the acoustic efficiency to the point that the sound radiated by free turbulence becomes negligible in comparison. This conclusion is still valid when the surface is not acoustically compact if the sound production is concentrated around a compact feature of the surface (e.g. close to an edge or an acoustically compact protuberance).

When a surface inside the region of aerodynamic sound production is not acoustically compact, its presence does not merely influence the aerodynamic field around it, but also the acoustic field. In this case, any solution of the acoustic field must satisfy the appropriate acoustic boundary condition on the surface. If the flow description corresponds to a compressible flow solution, the pressure on the wall satisfies the correct acoustic boundary conditions on the surface. However, if the flow description is incompressible, the correct acoustic boundary conditions cannot be enforced on the wall. As a result, Curle's analogy based on an incompressible flow pressure can yield significant errors for non-compact surfaces [2,3]. This has been highlighted, for instance, by computations of trailing edge noise when the airfoil chord is not acoustically compact [5–8].

* Corresponding author. Tel.: +32 16 384 498; fax: +32 16 384 350.
E-mail address: paula.martinez@lmsintl.com (P. Martínez-Lera).

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This difficulty can be circumvented by avoiding the use of formulations based on wall pressure. Instead, the acoustic problem can be solved numerically, with volume-distributed sources based on the velocity or vorticity field, by prescribing the correct hard-wall boundary condition on the surface. Alternatively, Green's function tailored to the surface can be used. Analytically tailored Green's functions are available for some simple geometries, such as a semi-infinite plane [9]. For instance, Wang et al. [10] applied this solution, with the extension proposed by Howe [11] for finite-length surfaces, to predict airfoil trailing edge noise. For complex geometries, tailored Green's functions can be obtained numerically; nevertheless, computing a tailored Green's function is in general not fundamentally easier than solving the acoustic boundary value problem numerically.

It may be speculated that when the pressure scattered by the surface is the strongest contribution to the acoustic radiation, information on the surface should suffice to characterize the sources, even when it is based on an incompressible flow description. This hypothesis is supported, for instance, by predictions of trailing edge noise on the basis of statistics of incompressible-flow (i.e., aerodynamic) surface pressure [12,13]. In these works, an analytical approach is applied to enforce the appropriate acoustic boundary conditions on the non-compact surface.

As an alternative to Curle's solution that retains the advantage related to defining the sources only on the surfaces, Schram [14] proposed a numerical approach in the framework of a direct boundary element method, in which the missing acoustic effects on the boundary are included by explicitly modelling the walls both as dipole sources of sound and as scattering surfaces for the sound radiated in the domain. This approach has some limitations. The distinction between acoustic scattering and sound production effects is, at best, ambiguous in the source region. In the approach proposed by Schram, this distinction is defined with the aid of a semi-empirical parameter that must be set by the user. Moreover, this concept is tailored to a specific direct boundary element method and is difficult to generalize to other approaches.

In this paper, a different approach is adopted. It is argued that the effect of dipole sources can be implemented through appropriate (Neumann) boundary conditions of an acoustic boundary value problem with an arbitrary geometry. A general formulation based on incompressible-flow wall pressure is derived analytically, and compared to Curle's solution.

The outline of the paper is as follows. Section 2 provides the theoretical background on which this work is based, including Lighthill's and Curle's analogies. Section 3 contains the derivation of the proposed formulation based on incompressible-flow pressure, including a theoretical analysis of the error compared to Curle's analogy. Section 4 shows a numerical test case as a proof of concept of the proposed approach, consisting of a trailing edge noise problem due to the flow past a slender body and, finally, Section 5 presents the conclusions of this study.

2. Theoretical background: Lighthill's and Curle's analogies

2.1. Lighthill's analogy and Curle's solution

Lighthill [1] was the first to propose an approach to quantify the amplitude of the sound produced by a flow. Lighthill's equation can be derived from the flow conservation equations, yielding

$$\frac{\partial^2 \rho'}{\partial t^2} - c_0^2 \frac{\partial^2 \rho'}{\partial x_i^2} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}, \tag{1}$$

where ρ' is the density fluctuation defined with respect to a reference density ρ_0: $\rho' = \rho - \rho_0$. T_{ij} is the Lighthill stress tensor, defined as

$$T_{ij} = \rho u_i u_j + (p' - c_0^2 \rho')\delta_{ij} - \tau_{ij}, \tag{2}$$

where τ_{ij} is the viscous stress tensor, u_i and u_j are components of the total velocity field and p' is the pressure fluctuation with respect to a reference pressure p_0: $p' = p - p_0$.

If the listener is surrounded by a uniform stagnant medium, and the constants c_0, p_0 and ρ_0 are interpreted, respectively, as the speed of sound, pressure and density of the undisturbed region where the listener is located, the left-hand-side term of Eq. (1) describes the acoustic propagation in the listener's region, while the right-hand-side represents an equivalent acoustic source.

For high Reynolds number flows, it is common to neglect the viscous contribution τ_{ij} with respect to the inertial term $\rho u_i u_j$. A recent work [15] shows that this assumption may be inaccurate for non-rigid surfaces with dimensions comparable to the viscous penetration depth, but this situation is out of the scope of the present work. For isentropic flows, the contribution $(p' - c_0^2 \rho')$ vanishes. Under these conditions, the density variations in the source region are of order M^2 (with the Mach number M defined as $M = U/c_0$, where U is the reference flow velocity). Therefore, assuming incompressible flow in the source region, it holds that $\rho u_i u_j \approx \rho_0 u_i^{(0)} u_j^{(0)}$, where $u_i^{(0)}$ and $u_j^{(0)}$ are components of the incompressible approximation of the velocity field. With these assumptions, Lighthill's tensor can be approximated as $T_{ij} \approx \rho_0 u_i^{(0)} u_j^{(0)}$, and Lighthill's equation (1) becomes

$$\frac{\partial^2 \rho'}{\partial t^2} - c_0^2 \frac{\partial^2 \rho'}{\partial x_i^2} = \frac{\partial^2 \rho_0 u_i^{(0)} u_j^{(0)}}{\partial x_i \partial x_j}. \tag{3}$$

It should be noted that the source term on the right-hand side of Lighthill's equation behaves as a quadrupole source.