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A load identification method based on wavelet multi-resolution analysis

Zong Li [a], Zhipeng Feng [b], Fulei Chu [a,*]

[a] Department of Mechanical Engineering, Tsinghua University, Beijing 100084, China
[b] Institute of Vehicular Engineering, University of Science and Technology Beijing, Beijing 100083, China

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ABSTRACT

Load identification, as a kind of indirect identification method, uses system characteristic and responses to calculate loads. A method based on wavelet multi-resolution analysis is proposed in this paper. By wavelet decomposition and transform at certain resolutions, the proposed method transforms the convolution relation between responses and loads in time domain into the linear multiplicative relation between system responses and wavelet responses in the wavelet domain. Loads can be identified as long as the linear multiplicative relation is solved. Qualitative and quantitative rules are proposed for selecting parameters that affect the accuracy of the proposed method, and are illustrated via numerical investigations. The method is illustrated by a multi-input-multi-output numerical simulation. A multi-input-multi-output laboratory experiment is performed to compare the proposed method with the frequency method on the identification ability.

1. Introduction

With the increase in engineering requirements and the improvement of engineering technology, one kind of problem regarding the determination of the exerted loads on a dynamic system urgently needs to be solved. However, in many practical problems, dynamic loads, such as aerodynamic loads exerted on aircrafts, wind loads applied on ocean platforms, the interaction between road surface and running tires and so on, are very difficult to be directly measured owing to technical or economic limitations. Load identification, also known as load reconstruction or load deconvolution in some literatures, uses system characteristics and responses to calculate loads. Compared with direct measurement, it is an indirect identification method. It can be used to calculate loads in severe environments and loads that cannot be acquired by transducers because the laying will change system characteristic or block the motion of system components.

Frequency method is one of the most commonly used identification methods. It uses a system's Frequency Response Function (FRF) and response spectrums to calculate load spectrums, and then derives loads in the time domain via the inverse Fourier transform. Some studies [1–3] determined forces in high-speed compression tests and in an experimental helicopter model with this method. However, references [4–6] found that it was very sensitive to the ill-condition of FRF and data noise. Using mathematical regularization technology, the identification ability of the method was improved [7–9]. Liu [10] analyzed two commonly used regularization methods: the truncated singular value decomposition method (TSVD) and the Tikhonov filter method in detail. Some advantages and disadvantages of the frequency method can be concluded from many literatures: owing to the linear multiplicative relation between responses and loads in the frequency domain, the

* Corresponding author. Tel.: +86 10 6279 2842; fax: +86 10 6278 8308.
E-mail address: chufl@mail.tsinghua.edu.cn (F. Chu).

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identification problem can be solved easily. However, the use of the Fourier transform requires an adequate data length. Inverse Fourier transformation must be performed at every discrete frequency and problems with numerical instability may arise when the frequency approaches the resonance frequency. Besides, ill-condition matrix and noise affect the identification accuracy severely.

In contrast to the Fourier transform, which expresses a function by a linear combination of sinusoidal functions having infinite duration, the wavelet transform expresses a function as a linear combination of wavelets that have finite durations. This is one of the most important features of wavelet transform [9]. Doyle [11] transformed a time domain convolution into the wavelet domain product to solve the identification problem, which started the application of the wavelet transform in load identification. However, the functions of wavelets were not fully exploited because the scale factor was fixed; thus, the proposed method was a single-resolution one-dimension wavelet transform in essence. Mao [12] used a similar method to identify single-input impulse load. In the loading phase, the result matched the original load well, whereas in the unloading phase there was a big oscillation. Nordberg [13] parameterized road profile into coefficients by the wavelet transform method to reduce the number of unknowns. Practical data was used to validate the proposed method and the results were satisfactory. However, due to the use of iteration, the method needed initial values that were difficult to be set. Being similar to the wavelet basis functions, Gunawan [22–24] used harmonic functions, quadratic splines functions and B-splines functions as the basis functions to approximate forces, especially impact forces. Besides, the ill-posed problem and regularization were discussed in these papers. Liu [25] used basis functions that spanned only the forcing space to reconstruct distributed forces. The method required fewer basis functions.

A novel load identification method based on wavelet multi-resolution analysis using the Impulse Response Function (IRF) of the system is proposed in this paper. The proposed method makes full use of the excellent features of the wavelet, such as adjustable time-frequency resolution, and several other advantages, such as that the stationary white noise transformed by orthogonal wavelet is still stationary white noise. The scale factor is adjustable in this method and this without using the iteration algorithm. The initial values do not need to be confirmed in advance. This method decomposes and transforms signals by two-dimension wavelets at a certain resolution and the convolution relation between responses and loads in time domain is transformed into the linear multiplicative relation between system responses and wavelet responses in the wavelet domain. The physical significance of the proposed method is explained during derivation. Through numerical analyses, qualitative and quantitative rules are proposed for selecting parameters that affect the accuracy of this method. The performance of the method is illustrated by a multi-input-multi-output (MIMO) numerical simulation. A MIMO laboratory experiment is also performed to compare the proposed method with the frequency method regarding identification ability. The results show that the proposed method has better identification ability and lower noise sensibility.

2. Load identification based on wavelet multi-resolution analysis

2.1. Daubechies wavelet

This method adopts *Daubechies* (db) wavelets as the basic wavelets. db Wavelets, which have some excellent features, such as orthogonality and compact support, were created and used by the Belgian scholar Daubechies in the 1990s. db Wavelets are named db$N$, where N represents the order. In all the interesting examples, the orthonormal wavelet bases can be associated with a multi-resolution analysis framework [20]. The concept of the multi-resolution analysis was introduced by Mallat in [21]. db Wavelets are orthogonal wavelets, which mean at a level (or scale) j of the wavelet basis function $\phi(t)$, there is

$$\int_{-\infty}^{\infty} \phi(2^j t - k_1)\phi(2^j t - k_2)\,\mathrm{d}t = \delta(k_1 - k_2), k_1, k_2 \in Z. \tag{1}$$

db Wavelets are compactly supported in $[0, 2N-1]$. Furthermore, for level j and shift factor k, the support interval is $[2^{-j}k, 2^{-j}(k+2N-1)]$.

2.2. Method deduction

In time domain, system responses are determined by the convolution of inputs and system characteristic

$$y(t) = h(t)f(t), \tag{2}$$

where $y(t)$ is the system response, $h(t)$ is the unit impulse response function and $f(t)$ is the load. Decomposing $f(t)$ with the wavelet yields

$$f(t) = \sum_{k=I}^{J} \alpha(k) 2^{j/2} \phi(2^j t - k) = \sum_{k=I}^{J} P(k)\phi(2^j t - k), \tag{3}$$

$$P(k) = \alpha(k) 2^{j/2}, \tag{4}$$

where $\alpha(k)$ are the scale coefficients, $P(k)$ are the weight coefficients, I and J are lower and upper limits of the integral, respectively (the difference is the number of weight coefficients), j is the decomposition level, k is the shifting factor and