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Simplified description of out-of-plane waves in thin annular elastic plates

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ABSTRACT

Dispersion relations are derived for the out-of-plane wave propagation in planar elastic plates with constant curvature using the classical Kirchhoff thin plate theory. The dispersion diagrams and the mode shapes are compared with their counterparts for a straight plate strip and the role of curvature is assessed for plates with unconstrained edges. Elementary Bernoulli–Euler theory for a beam of rectangular cross-section with the circular shape of its axis is also employed to analyze the wave guide properties of this structure in its out-of-plane deformation. The applicability range of the elementary beam theory is validated. The wave finite element method in the formulation of the three-dimensional elasticity theory is used to ensure that the comparison of dispersion diagrams is performed in the frequency range, where the classical thin plate theory is valid. Thus, the paper summarizes the effects brought to the propagation of out-of-plane waves in thin elastic plates by their constant curvature and the models of these plates.

1. Introduction

In various technical applications, it is necessary to investigate the waveguide properties of curved thin elastic strips. Such a strip may be curved spatially in the helical shape, or in the cylindrical shape, or in the shape of an annular plate. In each of these cases, the curvature influences the location of dispersion curves, so that the length and the mode shape of a free wave in the curved strip differ from their counterparts for a straight one with the same parameters.

Analysis of the out-of-plane elastic wave propagation in annular plates, possibly, resting at the elastic foundation, is relevant for dynamics of tracks and roadway pavements on large bridges. In both cases, the curved strip is an element of the compound waveguide, which, for example, may consist of two straight plates connected via a segment of the annular plate. A wave travelling in such a structure passes the curved segment, and the reflection/transmission phenomena at the interfaces are controlled by the mismatch between wavenumbers in neighboring segments.

These phenomena should also be considered with respect to the in-plane waves. In a planar compound structure as described above, the out-of-plane wave motion is not coupled with the in-plane one. The curvature-induced phenomena related to transmission of the in-plane waves in a thin elastic plate are captured by the model of an elastic layer in the plane stress state. Formally, the plane stress problem formulation requires re-definition of the speed of dilatation waves as compared with the plane strain. The comparison of dispersion diagrams for straight and curved elastic layers in the latter

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case has been performed in [1], and relatively weak influence of the curvature on dispersion diagrams has been reported. So, it is reasonable to expect that the role of curvature in dispersion of the in-plane waves in a thin plate is the same as in the elastic layer under plane strain conditions. To the best of the authors' knowledge, waveguide properties of annular and straight plates in the out-of-plane motion have not been compared, and this issue is addressed in the present paper. The references [2] and [3] present the dispersion diagrams for plates of these two types, but no comparison has been made. Furthermore, in the reference [2] an approximate solution in power series in the radial coordinate is used.

Closely related, but essentially different free vibration of complete annular plates is a very well established, if not trivial, subject covered in any textbook on the theory of vibrations (see, for example, [4,5]). The obvious assumption employed in its analysis is the harmonic decomposition of the displacement field in the circumferential direction. The solution in the form of standing wave is assumed, and the spectrum of eigenfrequencies is found for each number of circumferential waves. The same methodology is perfectly valid for analysis of vibration of sector annular plates. The modern tools of computations (the finite element method) embedded in the commercially available software make it a straightforward exercise to calculate eigenfrequencies and eigenmodes of these structures. The survey of papers concerned with standing waves in complete or sector annular plates lies beyond the scope of the present paper. Likewise, this paper is not concerned with the survey of literature on wave propagation in a straight elastic layer (known as Rayleigh–Lamb problem), or in an elastic cylindrical shell. These classical issues are covered in, for example, [6–8]. Free wave propagation in an infinite straight simply supported plate strip has been considered, for example, in [9]. Much less attention has been paid to the curved waveguides. Acoustic wave propagation in curved ducts has been considered in [10,11]. The comparison of dispersion diagrams for a curved and a straight elastic layer with various boundary conditions has been done in [1].

Addressing the theory of elastic wave propagation in circular curved beams, it should be noted that the in-plane problem formulation has gained much more attention in the literature, than the out-of-plane one. In particular, the detailed study of the in-plane power transmission has been presented in [12,13] in the framework of classical beam theory. The latter paper is also concerned with the matrix representation of the in-plane wave motion. In the recent paper [14], alternative models are introduced to analyze coupling of in-plane flexural, tangential and shear modes in standing waves of circular curved beams. Out-of-plane vibrations of curved beams with circular or tubular cross-section have been considered in, for example, [15] and [16]. These papers are concerned with the standing waves in circular rings described within Bernoulli–Euler or Timoshenko beam theories. The out-of-plane elastic wave motion in circularly curved beams of non-circular cross-section has not yet been fully addressed in literature.

The paper is structured as follows. First, an annular plate with unconstrained edges is considered as a waveguide and the location of dispersion curves is compared with the location of dispersion curves for an infinitely long straight strip with the same boundary conditions. In Section 2.3, the comparison of the mode shapes is done both in the cases, when wavenumbers are very close to each other and when the difference is relatively large at the given excitation frequency. Section 3 is concerned with the analysis of dispersion diagrams in the 'low frequency—long wave' limit, where elementary Bernoulli–Euler theory may be used. In this section, the Timoshenko model of torsion stiffness of a beam with the rectangular cross-section is used, and its validity is assessed. The verification of results obtained by use of classical thin plate theory by means of the wave finite element method with solid elements in the ANSYS environment is presented in Section 4. The results reported in the paper are summarized in conclusions.

2. Out-of-plane wave propagation in straight and annular thin plates

2.1. Mathematical model

We consider a thin elastic strip of thickness h and width b, which is either straight or, as shown in Fig. 1, curved. In the latter case, the curvature is assumed to be constant. We are concerned with the wave motion, in which the out-of-plane displacement w (i.e., the displacement in the z-direction) is dominant. To perform the analysis of propagation of this type

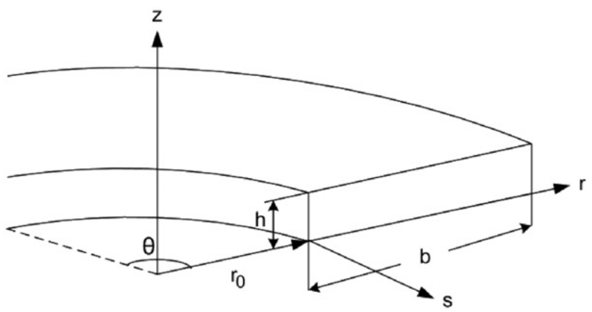

Fig. 1. Section of a curved thin annular plate.