Journal of Sound and Vibration 326 (2009) 870–882

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Suppression of time aliasing in the solution of the equations of motion of an impacted beam with partial constrained layer damping

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ARTICLE INFO

Article history:
Received 1 November 2008
Received in revised form
16 May 2009
Accepted 1 June 2009
Handling Editor: C.L. Morfey
Available online 16 July 2009

ABSTRACT

Signal processing issues encountered when an analytical spectrum is converted in the time domain with an inverse discrete Fourier transform are investigated. In the analytical model of the transient time response of an impacted beam with partial constrained layer damping (PCLD) developed in Ref. [D. Granger, A. Ross, Effects of partial constrained viscoelastic layer damping parameters on the initial transient response of impacted cantilever beams: experimental and numerical results, *Journal of Sound and Vibration* 321 (1–2) (2009) 45–64, doi:10.1016/j.jsv.2008.09.039], noncausal effects were observed for lightly damped structures. As discussed in the present paper, the noncausal effects were due to time aliasing occurring when continuous frequency spectra were discretized. To suppress such errors, the numerical Laplace transform is introduced and applied to the previous model, which was based on Fourier transforms. The equations of motion of the system and the viscoelastic properties of the core are formulated in the Laplace domain. A window is used in the Laplace domain to avoid amplification of the Gibbs oscillations that are caused by the truncation of the spectrum. The new solution technique is compared to the previous method. It is shown that noncausal effects appearing in the first milliseconds of time signals with the use of discrete Fourier transforms are avoided with the Laplace transform solution method. Numerical results are validated for transient responses using experimental impact force signals. The results are in good agreement with experimental data.

1. Introduction

Transient responses of impacted structures are a source of concern, especially when dealing with occupational health. Many manufacturing processes, such as riveting, are based on mechanical impacts. Strong impacts induce significant transient vibrations and powerful transient noises that can undoubtedly cause hearing impairment. Sound production is particularly strong with large, flexible impacted structures. Both the lack of effectiveness of auditory protection equipments and the reluctance of workers to use them can result in hearing injuries. Therefore, partial constrained layer damping

Abbreviations: a.u., arbitrary units; CLD, constrained layer damping; DFT, discrete Fourier transform; dof, degree of freedom; EOM, equations of motion; FFT, fast Fourier transform; FRF, frequency response function; NLT, numerical Laplace transform; PCLD, partial constrained layer damping.

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0022-460X/$ - see front matter
doi:10.1016/j.jsv.2009.06.001

(PCLD) is sometimes used to reduce vibrations and, thus, the radiated noise. PCLD consists of a viscoelastic layer topped with a constraining elastic layer that both cover part of the structure. PCLD can be applied to existing structures, and was shown to be quite efficient when compared to complete constrained layer damping (CLD) [1].

Impact noises are partly due to the initial transient motion of structures [2–4]. Damping treatments designed to reduce such vibrations would therefore greatly benefit from the modeling for transient time response of impacted mechanical structures. Granger and Ross presented an analytical model of the time domain dynamics of PCLD treated beams [5]. Beams were used as a basic structure to understand the behavior of more complex structures. In this model, the viscoelastic properties of the core were nonlinear and the shear modulus was modeled using a Prony series in the Fourier domain. The equations of motion (EOM) of the system were obtained using Lagrange's equations. The EOM were converted in the frequency domain using a Fourier transform, and were solved for frequency displacements using the assumed modes method. The displacements were then converted back in the time domain using inverse discrete Fourier transforms (DFT). The excitations used were experimental force signals. Time responses were shown to be in fairly good agreement with experimental data, for various PCLD parameters.

It was shown, however, that responses calculated using the analytical model and solution method were noncausal for systems with poor damping ratios. This was explained by the fact that the time signals for lightly damped beams remained significant at the end of the calculation period, causing time aliasing in the solution process.

In order to extend the applicability of the model to any damping ratio, the present paper discusses time aliasing and Gibbs oscillations, two phenomena encountered when analytical spectra are inverted numerically. The numerical Laplace transform (NLT) and windowing in the frequency domain are introduced to significantly reduce these signal processing issues. The model used by Granger and Ross is thus reformulated using a Laplace transform approach in order to avoid noncausal effects and produce reliable initial transient responses. The response can be used in parametric studies of PCLD treatments, regardless of the damping ratio of the system. Both formulations in the Fourier and the Laplace domains are compared using experimental force signals and different PCLD configurations. Finally, the model developed in the Laplace domain is validated experimentally.

2. Numerical inversion of an analytical spectrum

The method developed by Granger and Ross did produce interesting results, but lead to inaccuracies at the very beginning of the time response signals. The authors stated that these noncausal displacements resulted from the use of discrete Fourier transforms to invert the frequency response function (FRF) of PCLD beams. The magnitude of the error was significant for signals that were not sufficiently damped by the end of the observation window. Fig. 1 shows an example of the initial transverse displacement of an impacted beam for two configurations for which damping ratios differ. It is seen in this figure that the time signal corresponding to the beam configuration that is highly damped (——) is causal while the other signal (— - —) is not. Such problem occurred even though simulations were run over 26 s, which cost enormous computing time and memory allocation. It will now be shown how the noncausal displacements are created during the inverse discrete Fourier transform operation.

The inverse DFT is written as

$$f_n = \frac{\Delta\omega}{2\pi} \sum_{q=-N/2}^{N/2-1} F_q \, \mathrm{e}^{\mathrm{i}2\pi qn/N}, \tag{1}$$

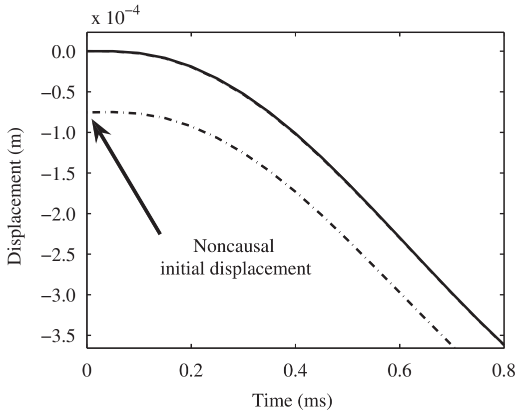

Fig. 1. Example of inaccuracy at the beginning of simulated beam displacements [5]: high (——) and low (— - —) damping.