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A numerical approach for the computation of dispersion relations for plate structures using the Scaled Boundary Finite Element Method

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ABSTRACT

In this paper, a method is presented for the numerical computation of dispersion properties and mode shapes of guided waves in plate structures. The formulation is based on the Scaled Boundary Finite Element Method. The through-thickness direction of the plate is discretized in the finite element sense, while the direction of propagation is described analytically. This leads to a standard eigenvalue problem for the calculation of wave numbers. The proposed method is not limited to homogeneous plates. Multi-layered composites as well as structures with continuously varying material parameters in the direction of thickness can be modeled without essential changes in the formulation. Higher-order elements have been employed for the finite element discretization, leading to excellent convergence for complex structures. It is shown by numerical examples that this method provides highly accurate results with a small number of nodes while avoiding numerical problems and instabilities.

1. Introduction

Over the last decades, Lamb waves, theoretically described in 1917 [1], have gained attention as a promising tool in non-destructive testing and structural health monitoring. Many attempts have been made to utilize Lamb waves for defect detection in large plates [2,3]. However, as Lamb wave modes are highly dispersive, they lead to complex signal interpretation, thus limiting practical applications. Additionally, the number of propagating modes depends on the excitation frequency and plate thickness as well as the material parameters. General remarks on the dispersion properties of Lamb waves can be found in [4].

Dispersion curves can be measured and compared with theoretical data to obtain the thickness variation of a plate or its material constants [5–7] and anisotropy effects [8,9] of plates. Concurrently, techniques have been studied to remove the effect of dispersion in structural health monitoring [10,11] and inspection [12–14] applications. Group velocity dispersion has been assessed using time–frequency analysis [15]. For all applications the accurate theoretical prediction of dispersion characteristics is essential.

For the case of a homogeneous plate of constant thickness, the dispersion curves can directly be obtained analytically, as described in detail in [16]. However, since similar guided waves can be used for complex structures, such as layered composites [17,18] and functionally graded materials, a general approach to obtain dispersion properties is desirable.

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Early approaches include the calculation of a lumped mass matrix of a layered structure to obtain dispersion relations of Rayleigh waves [19]. The discretization of structures with thin layers has led to eigenvalue problems to calculate of dispersion relations [20,21]. This so-called thin-layer method has also been applied to assess the impulse response of layered media in the time domain [22] and for the analysis of piezo-composite layers [23]. Stiffness methods have been used to obtain approximate dispersion equations of layered media [24] and, more recently, for the characterization of multidirectional composites [25].

The transfer matrix method [26,27] analytically describes the wave propagation in every layer of a multi-layered structure and thus can be used for the calculation of dispersion curves. Based on this idea, the global matrix method [28,29] has been developed to avoid the numerical problems mainly encountered at high frequency-thickness products in the transfer matrix method. The matrix methods have been extended to anisotropic cases [30] and successfully applied to the calculation of dispersion curves in orthotropic plates [31]. Matrix methods are discussed in [32]. Applications of these methods include the calculation of dispersion characteristics of different multi-layered structures [33–36]. The commercial software *disperse* [37], used for calculating dispersion relations for plates and cylinders and the free tool *PCDISP* [38], used for cylindrical waveguides make use of these matrix methods. Recently, a different approach utilizing the Green's matrix and applying Fourier transformation to derive a boundary problem for ordinary differential equations has been used to describe oscillation in multi-layered anisotropic composites [39].

To calculate the dispersion properties of plates with continuously varying material properties in the thickness direction, a power series technique has been employed to derive approximate analytical solutions [40].

In contrast to the analytical approaches previously mentioned, purely numerical methods have been applied to obtain dispersion curves for complex structures. These approaches are of interest especially when waveguides of arbitrary cross-section are to be studied rather than plates. Dispersion curves can, in some cases, be obtained using standard Finite Element software by discretizing a representative part of a three-dimensional waveguide [41]. In a different approach, a unit length section of a three-dimensional waveguide is assessed, reducing the problem to a two-dimensional mesh [42].

Wave propagation in composite plates has been studied by applying the Finite Element as well as the Boundary Element Method [43]. Dispersion curves have been obtained by discretizing only the through-thickness direction of the composite with linear or quadratic elements, leading to a quadratic eigenvalue problem for the wavenumbers for a given frequency. Similarly, wave propagation in anisotropic laminated strips [44], thin walled waveguides [45] and damped waveguides of arbitrary cross-section have been studied [46] by discretizing the cross-section only. This approach is also used in the so called semi-analytical finite element (SAFE) method which is based on [47] and [48] and has been applied to composite plates [49,50], anisotropic composite cylinders [51,52], axisymmetric damped waveguides [53], as well as rods and rails [54]. The software *GUIGUW* [55], which uses a SAFE formulation, is under development and free demo versions are already available. Recently, significant extensions of the SAFE formulation have been published for the computation of the transient response [56,57].

Additionally, special formulations have been employed for cylindrical waveguides of functionally graded materials [58] and functionally graded piezoelectric plates [59], assuming the material properties vary linearly within each element. This formulation has been extended to cylinders of piezoelectric materials [60].

In this paper the formulation of the Scaled Boundary Finite Element Method (SBFEM) is applied for the calculation of dispersion characteristics in plate structures. Generally, the SBFEM is a semi-analytical method and requires discretization of the boundary only [61,62]. It is advantageous in modeling wave propagation, especially in cracked structures [63]. For the present application the procedure can be simplified to the discretization of a single straight line. The plate can consist of an arbitrary number of layers or a material with continuously varying elastic parameters. This new approach leads to a quadratic eigenvalue problem for the calculation of wave numbers, similar to those obtained by the so-called SAFE formulations previously mentioned. In this work, a standard eigenvalue problem has been derived by solving for both displacements and nodal forces simultaneously, significantly decreasing computational costs in comparison with the polynomial eigenvalue problem. For the discretization of the through-thickness direction higher-order spectral elements have been employed in this work. This drastically reduces the number of nodes required to obtain accurate solutions in comparison with linear or quadratic elements used in previous work. The use of spectral elements additionally reduces the number of non-zero entries in the eigenvalue problem. Consequently, the solution can be obtained very efficiently even for comparably high frequencies. This method is particularly advantageous for complex functionally-graded materials. The application of higher-order elements allows the material properties to be described by complex functions (e.g. polynomials up to the same order as the element shape functions) with a small number of nodes in comparison with linear or quadratic elements.

The paper is organized as follows. The next section describes the governing equations and the scaled boundary formulation for this particular problem of wave propagation in plates. In Section 3, the use of high-order elements is summarized. Thereafter, several implementation considerations are explained in Section 4. In Section 5 numerical examples for different plate structures are compared with analytical solutions and different numerical results presented in the literature. A conclusion of the present work is given in Section 6.

2. Scaled Boundary Finite Element formulation for propagating modes in plates

The plate of constant thickness h shown in Fig. 1 is addressed. Without loss of generality the y-coordinate of a Cartesian coordinate system is chosen along the through-thickness direction of the plate while the x-coordinate is parallel to the