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Exact 3D elasticity solution for free vibrations of an eccentric hollow sphere

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ABSTRACT

An exact three-dimensional elastodynamic analysis for describing the natural oscillations of a freely suspended, isotropic, and homogeneous elastic sphere with an eccentrically located inner spherical cavity is developed. The translational addition theorem for spherical vector wave functions is employed to impose the zero traction boundary conditions, leading to frequency equations in the form of exact determinantal equations involving spherical Bessel functions and Wigner 3j symbols. Extensive numerical calculations have been carried out for the first five clusters of eigenfrequencies associated with both the axisymmetric and non-axisymmetric spheroidal as well as toroidal oscillation modes for selected inner-outer radii ratios in a wide range of cavity eccentricities. Also, the corresponding three-dimensional deformed mode shapes are illustrated in vivid graphical forms for selected eccentricities. The numerical results describe the imperative influence of cavity eccentricity, mode type, and radii ratio on the vibrational characteristics of the hollow sphere. The existence of "multiple degeneracies" and the trigger of "frequency splitting" are demonstrated and discussed. The accuracy of solution is checked through appropriate convergence studies, and the validity of results is established with the aid of a commercial finite element package as well as by comparison with the data in the existing literature.

1. Introduction

The dynamic behavior of shells has been a subject of study for more than a century, since they are extensively used as one of the basic components in civil, mechanical, aircraft, and naval structures. Many of these studies are based on classical or thin-shell theories, which utilize the well-known simplifying Kirchhoff–Love postulates of straight inextensional normal, making these theories inadequate for the analysis of thick shells. In recent years, the refinement of thin-shell theories has resulted in a number of the so-called higher order shell theories based on various approximations of shear deformation and rotary inertia associated with the transverse direction of the shell surface [1–3]. For dynamic analysis of complex structures, thin or higher order shell theories are most appropriate, but they lose their accuracy for thick structures, making the use of three-dimensional theory of elasticity inevitable. The high computational power requirements being the only limiting factor, the three-dimensional theory of elasticity may advantageously be used to accurately extract the full vibration spectrum of natural frequencies and mode shapes for simple geometries such as the solid or hollow elastic spheres and cylinders without any missing modes. Such analysis, not only provides reliable solutions for benchmarking purposes, but also can bring out the physical characteristics of the problem.

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Natural vibration of freely suspended solid and thick hollow elastic spheres has long been one of the fundamental problems in elastodynamics, originally associated with interest in the oscillations of the earth [4]. Numerous researchers have carried out three-dimensional analysis based on the linear equations of elasticity in order to find accurate natural frequencies for the vibrations of solid or hollow thick spheres. Restricting our attention to the purely isotropic problems, the most important works relevant to the present study will briefly be reviewed here. The first known complete mathematical treatment for the vibrations of an ideal isotropic elastic sphere was given by Lamb [5] in Cartesian coordinates and by Chree [6] in the more convenient spherical coordinates. In particular, Lamb [5] presented two basic types of free vibrations: torsional (toroidal or equivoluminal) vibrations or vibrations of the first class associated with the rotatory motions of the sphere where there is no change in volume of the sphere and no radial displacement; and spheroidal (pulsating or coupled bending–stretching) vibrations or vibrations of the second class which are related to the distortion of the elastic sphere due to vibrations in the radial direction. Most of the subsequent studies were based on the work of Lamb and driven by the interest in understanding the generation of earthquakes and their effect on earth. For instance, Sato and Usami [7,8] performed a detailed spectral analysis and provided (tabulated) a comprehensive set of results for the frequencies of free vibration and their corresponding displacement field distributions within vibrating homogeneous elastic spheres. Subsequently Shah et al. [9] studied the free vibrational behavior of hollow spheres by using an exact three-dimensional analysis to obtain a characteristic equation solvable in terms of spherical Bessel functions of the first and second kind and gave numerical results for a wide range of thickness to radius ratios in graphical form. Extensive work on the vibration of spheres including complicating effects, such as anisotropic material properties, liquid core, multi-layers and self-gravitation effects, has been carried out by numerous researchers. An excellent reference on the subject is the very readable book by Lapwood and Usami [10]. Hosseini-Hashemi and Anderson [11] examined issues of normalization of the torsional vibration characteristics of solid elastic spheres and presented three-dimensional diagrams for the surface displacement of the toroidal modes. Heyliger and Jilani [12] used the Ritz method to verify the results of Sato and Usami [7,8] for isotropic spheres and gave some results for inhomogeneous spheres as well as an excellent list of references. Kumbasar [13] used three-dimensional elastodynamic equations to study free vibrations of complete thick spherical shells. Chang and Demkowicz [14] determined natural frequencies of a vibrating hollow, elastic sphere using both the 3D elasticity and Kirchhoff shell theory. McGee and Spry [15] used a complete set of algebraic trigonometric polynomials to approximate the radial, meridional, and circumferential displacements (via Ritz method) to address the spheroidal and toroidal elastic vibrations of thick-walled, spherical bodies of revolution based on the three-dimensional theory of elasticity in curvilinear coordinates. Chau [16] used two scalar wave potentials to derive the exact frequency equation and numerical results for only the toroidal mode of vibrations for a spherically isotropic elastic sphere. Buchanan and Rich [17] formulated a finite element model to study the effects of the boundary conditions on free vibration of thick isotropic spherical shells. Saviot and Murray [18] presented a qualitative distinction between different spheroidal modes of vibration of a free continuum elastic sphere. Duffey et al. [19] compared natural frequencies and mode shapes obtained from axisymmetric and non-axisymmetric theories of vibration of complete free spherical shells with those from finite element computer simulations with and without geometrical imperfections.

Analytical solutions of interior or exterior boundary value problems in different fields such as potential theory, acoustics, elastodynamics, and electromagnetism are stringently dependent on the shape of boundaries. In particular, when multiple (eccentric) interfaces are present in a wave field, there is an interaction between them due to cross scattering. The theoretical basis for the solution of the wave interaction problems involving eccentric spherical boundaries was set by Friedman and Russek [20], Stein [21] and Cruzan [22], who provided the translational addition theorems for spherical wave functions. After that, several researchers have studied the eccentric sphere problem. Among them, Golovchan [23] considered axisymmetric forced oscillations of (dynamic stress concentrations within) an elastic sphere with a non-concentric inner cavity under the action of a uniform external pressure. Kanellopoulos and Fikioris [24] obtained the natural frequencies for the scalar interior boundary value problem in the acoustic region between two eccentric spherical surfaces, for both Neumann and Dirichlet boundary conditions. Roumeliotis et al. [25] and Roumeliotis and Kanellopoulos [26] employed a special shape perturbation method to derive analytical expressions for the acoustic resonance frequency shifts in a hard- (soft-) walled spherical cavity, caused by introduction of an eccentric small inner sphere. Roumeliotis et al. [27] used spherical vector wave functions and related addition theorems to derive analytical expressions for the resonant frequencies in an electromagnetic spherical cavity with an eccentric inner electrically small and perfectly conducting sphere for both the magnetic and electric modes. Roumeliotis et al. [28] used the classical method of separation of variables combined with translational addition theorems for spherical vector wave functions to treat the scattering of a plane electromagnetic wave from a metallic or dielectric sphere of electrically small radius, embedded into a dielectric one. Cottis and Moyssidis [29] investigated the effect of eccentricity on the complex resonances of a conducting sphere eccentrically coated by a dielectric one. Charalambopoulos et al. [30] used the elasticity theory in conjunction with the bispherical coordinates system to study the effect of the thickness nonuniformities of the human dry skull (modeled by an isotropic elastic material occupying the region bounded by two non-concentric spheres) on its frequency spectrum. Their analysis is however independent of the azimuthal coordinate (i.e., perfectly axisymmetric), and is only valid for small deviations of the system from the concentric case. Ioannidou and Chrissoulidis [31] obtained an exact solution to the problem of electromagnetic-wave scattering from a sphere with an arbitrary number of non-overlapping spherical inclusions by use of the indirect mode-matching technique. Videen [32] derived the scattered fields from a spherical body eccentrically located within an otherwise homogeneous host sphere due to an arbitrarily positioned seismic source,