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Optimal steady-state design of zone volumes of bioreactors with Monod growth kinetics

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ABSTRACT

This paper deals with steady-state analysis and design of bioreactors consisting of a number of completely stirred tank reactors (CSTRs) in series. The study is confined to one consumed (substrate) and one consuming constituent (biomass). The specific microbial growth rate is assumed to be described by Monod kinetics. The death of biomass is assumed to be negligible. Two optimal design problems for a large number of CSTRs in series are studied: to minimize the effluent substrate concentration for a given total volume, and to minimize the total volume for a given effluent substrate concentration. As an appealing alternative to solve these problems numerically, it is proposed to consider the asymptotic case where the number of CSTRs tends to infinity. This is shown to correspond to one CSTR in series with a plug flow reactor (PFR). A CSTR with a sufficient large volume is needed to avoid wash-out of the biomass. The main result is that both design problems for the CSTR + PFR configuration have the same solution with respect to the optimal volume of the CSTR, which is given as an explicit function of the incoming substrate concentration, the volumetric flow rate and the coefficients of the Monod growth rate function. Numerical results indicate that the plug flow approach may be used as a feasible design procedure even for a reasonably low number of CSTRs in series.

1. Introduction

Bioreactors are of fundamental importance in industrial applications. Usually the goal is to reduce the substrate concentration of a certain flow by passing it through one or several bioreactors in series [1]. The bioreactors are typically modeled as completely stirred tank reactors (CSTRs) containing microorganisms (biomass) which grow through consumption of the substrate, where the Monod kinetics are often used to describe biological treatment processes (see, for example, Monod [2], Koch [3]). A natural request is to optimize the process in terms of volumes and performance.

Optimal design of bioreactors has been extensively studied during the last 50 years. From the early contribution presented by Aris [4], important efforts have been made to design general methods for analyzing and optimizing CSTR volumes. As for CSTR analysis, Herbert [5] studied the behavior of two-reactor cascades for the case of single-stream and multiple-stream assuming that there was no death of microorganisms. For the CSTR optimization, the typical approach has been to find the optimal distribution of volumes for a given (upper) limit of the effluent substrate concentration (see, for example, [6]). In these studies, the term "optimal" refers to the minimum of the total volume of CSTRs required to satisfy the effluent restriction.

Early studies about CSTR optimization can be found in Bischoff [7], where the optimization of two CSTRs in series with a single stream was investigated. The aim was to minimize the total residence time for a required effluent assuming no death rate of microorganisms. Grady and Lim [8] showed graphical examples of equal and non-equal size CSTRs in series obtained by quantifying the total space (residence) time (i.e., total volume divided by flow rate), modeling by Monod kinetics and assuming death of microorganisms. Although no mathematical proof was presented, the examples showed that configurations with the majority of the total volume in the first CSTR give a lower substrate reduction than configurations with a total volume equally distributed. The study also indicated that no generalizations can be made about the best volume distribution, since it will depend on the desired effluent concentration.

Braha and Hafner [9] reported graphical solutions, referred as nomograms, to characterize and design multi-stage CSTRs. These nomograms allow the dimensioning of the process and the deter-

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mination of some kinetic parameters. Harmand et al. [10] solved the problem of two CSTRs considering a distributed feeding system and/or a recirculation loop. Nelson and Holder [11] presented asymptotic solutions for the effluent concentration problem of up to $N = 4$ CSTRs in series and assuming the same volume in the CSTRs. The process was modeled by Contois kinetics [12] and two scenarios were studied. In Scenario 1, the total residence time (τ_t) is assumed to be given, and Scenario 2 considers that τ_t is slightly larger than the wash-out condition. Results from Scenario 1 show that the effluent decreases like $(\tau_t)^{-N}$, conjecturing that this relationship holds for all N. Scenario 2 shows that a small increase in τ_t leads to a large decrease in the effluent.

Optimal design of CSTRs in series has in literature been compared to the design of a plug flow reactor (PFR). Luyben and Tramper [13] derived an analytical expression for the optimum (defined as the minimum total reactor size) to perform a specific conversion for a reactor modeled by Monod kinetics. Some numerical examples were shown for up to 10 reactors in series in two cases: optimum volumes and equal-sized volumes. For comparison, the case of a PFR was also evaluated. Results show that the difference in the volume of the first two CSTRs is larger for low values of N, and becomes smaller as N increases. They also show that the performance of N CSTRs converges to one CSTR followed by a PFR as N increases. Based on the critical substrate concentration concept, Hill and Robinson [14] derived an expression for the minimum possible total residence time for a series of N CSTRs. Expressions were derived for several growth kinetics and illustrated for Monod kinetics. A comparison with a PFR was also included.

Using the critical substrate concentration concept [14], Gooijer et al. [1] presented an optimization criterion based on the total holding time in order to decide if and when CSTRs in series are superior to a single CSTR. The study included the case of a single CSTR followed by a PFR. Expressions for the minimum holding time for the case of one and two CSTRs in series were derived assuming different kinetics.

CSTRs and PFRs are frequently used for wastewater treatment applications; see e.g. [15,16]. In this process the bioreactors form a configuration known as activated sludge process (ASP), where the bioreactor is followed by a settler (which increases the microorganism concentration) and a recycle stream. Optimal design of these bioreactors has been extended to ASP. For example, Erickson and Fan [17] applied mathematical optimization techniques in an ASP modeled by Monod kinetics and formed by up to three CSTRs in order to solve two cases. In Case I, the total volume is minimized for a fixed effluent, and in Case II, a cost function with prices assigned to the organic waste being discharged and to the total volume is minimized. Numerical examples showed that the minimum cost of a two-tank and a three-tank system is between 60–70% and 50–60% of the one-tank system, respectively. Scuras et al. [18] suggested that an ASP with three CSTRs is generally the best choice for a system modeled by Monod kinetics (an additional tank is unlikely to provide significant economic payoff), and that non-equal size volumes are the optimal design. Ali San [19] studied the design of a PFR in a simple ASP modeled by Monod kinetics and including a death rate of biomass for a desired effluent substrate concentration. An implicit and approximate expression for the effluent substrate was derived and compared with numerical solutions.

The recycle stream has also been considered for solving the optimal design of ASPs, see e.g. [17,20–22]. In particular, Alqahtani et al. [20] analyzed an ASP with one CSTR when two and three ASPs are disposed in series; thus the recirculation of the ith settler is added as input to the ith bioreactor. Different configurations for recirculation were considered in order to determine the minimum effluent. Contois growth kinetics and equal-sized volumes were assumed. They found that an optimal design exists if perfect (full volumetric flow) recycle is applied to the final bioreactor and imperfect (fraction of the volumetric flow) recycle is applied to the rest of bioreactors. Implications for the optimal solutions regarding short and large total retention time were also detailed.

The effect of the retention time was also analyzed by Alqahtani et al. [21] for optimizing an ASP with N bioreactors in series ($N = 2, \ldots, 5$). In that study, the recycle was placed to the first bioreactor. Again, Contois kinetics and equal-sized volumes were considered. Results show the effect of the settler when the total residence time is larger or lower than a certain critical value. Also, it was analyzed in [21], in the case of two bioreactors in series and a given effluent restriction, whether it is advantageous to add an extra bioreactor or a settler. In all references above, the settler has been assumed to work ideally. However, it is known that the nonlinear continuous sedimentation process in the settler influences the steady states of the ASP in a complicated way already when only one CSTR is present; see [23].

Recently, Zambrano and Carlsson [24] presented a study about the optimal design of CSTRs in series. In this study the optimal design of volumes was constrained by a given total volume, and the aim was to find the optimal volumes so that the effluent substrate concentration in steady state is minimized. Both Monod and Contois kinetics were used for describing the growth kinetics and assuming no death of microorganisms. Numerical results for the cases $N = 2$ to $N = 5$ CSTRs in series show that the optimal volumes are very different depending on the choice of the kinetics. Sidhu et al. [25] presented a steady-state analysis of CSTRs in series with single and multiple feed streams. The reactors were modeled by Monod kinetics and were assumed to have equal volumes. One result was that a configuration with equally distributed feed streams is not better than a single CSTR.

The aim of the present paper is to present new approaches in the optimal design of bioreactors when the number of CSTRs is large. We assume that the biomass and substrate follow the Monod kinetics and the death of the biomass can be neglected. We present new approaches for optimal bioreactor design for two scenarios: (i) find the minimum effluent substrate concentration for a given total reactor volume; and (ii) find the minimum total reactor volume for a given effluent substrate concentration. In particular, analytical expressions are derived for solving these two design cases.

The paper is organized as follows: First, a system of N CSTRs in series is presented. Next, a model of the system based on ordinary differential equations is revisited. Then, the new approaches are detailed, and several numerical examples are used for illustration. Finally, discussions and conclusions are drawn.

2. Materials and methods

2.1. N CSTRs in series

Consider a bioreactor volume V formed by N CSTRs in series as shown in Fig. 1.

Each CSTR has the same and simple growth reaction (one growth limiting substrate, one biomass). The Monod kinetics are used as the specific growth rate of the biomass used for different demonstrations and examples in this study. This growth rate is given by

$$\mu(S_i) = \mu_{\max} \frac{S_i}{K_s + S_i} \tag{1}$$

where $\mu_{\max}$ is the maximum specific growth rate, and K_s is the half saturation constant, i.e., the substrate concentration at which the rate is half the maximum. The effluent volumetric flow rate is equal to the influent flow rate Q. S_{in} is the influent substrate concentration. It is assumed that no biomass is present in the influent ($X_{\text{in}} = 0$). The time-dependent ordinary differential equations describing the dynamics of the substrate and biomass concentra-