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An efficient realization of frequency dependent boundary conditions in an acoustic finite-difference time-domain model

José Escolano[a,b,*], Finn Jacobsen[a], José J. López[b]

[a]Acoustic Technology, Ørsted.DTU, Technical University of Denmark, Building 352, DK-2800 Kgs. Lyngby, Denmark
[b]Institute of Telecommunications and Multimedia Applications, Technical University of Valencia, E-46022 Valencia, Spain

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Abstract

The finite-difference time-domain (FDTD) method provides a simple and accurate way of solving initial boundary value problems. However, most acoustic problems involve frequency dependent boundary conditions, and it is not easy to include such boundary conditions in an FDTD model. Although solutions to this problem exist, most of them have high computational costs, and stability cannot always be ensured. In this work, a solution is proposed based on "mixing modelling strategies"; this involves separating the FDTD mesh and the boundary conditions (a digital filter representation of the impedance) and combining them into a global solution. This solution is based on an interaction model that involves wave digital filters. The proposed method is validated with several test cases.

1. Introduction

Numerical calculations of sound fields in complex enclosures must take account of aspects as for example geometric shape and properties of materials. The current power of computers makes it possible to approach the solution of such problems with several methods. One important time-domain method is known as the finite-difference time-domain (FDTD) method [1,2]. This method provides a simple and accurate solution with relatively low computational cost. However, one of its handicaps is that it is fairly complicated to take account of frequency dependent complex impedance boundary conditions, which is important, e.g., in room acoustics [3] and in outdoor sound propagation [4]. A review of time-domain impedance boundary conditions can be found in a paper by Fung and Ju [5].

Most time-domain impedance models are based on a modification of the wave equation at the mesh points where the impedance is situated [6,7]; and to define a general method for any analytical expression for the impedance and ensure its stability is not a simple task. The method presented in this paper combines an FDTD mesh with a digital filter representation of the boundary condition. This combination of simulation methods is

*Corresponding author at: Institute of Telecommunications and Multimedia Applications, Technical University of Valencia, E-46022 Valencia, Spain.
E-mail addresses: escolano@ujaen.es (J. Escolano), fja@oersted.dtu.dk (F. Jacobsen), jjlopez@dcom.upv.es (J.J. López).

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known in some contexts as ‘‘mixing modelling strategies’’ [8]; this means that a combination of method can improve the entire method. However, as will become clear, this combination cannot be made directly because of stability problems. This will be solved by using a wave digital filter (WDF) as a common interface [9]. For simplicity, the theory presented in this paper is for 2D problems. However, it is straightforward to extend it to 3D.

2. FDTD theory

The FDTD method, as first proposed by Yee [10], is a simple and elegant way of discretizing the differential form of Maxwell's equations. In FDTD modelling of wave equations, the space solution is discretized using the ‘‘Yee cell’’, and the vector component of the electrical and magnetic field are distributed around the unit cell so as to allow the differential operators to be approximated by second-order centred finite differences that combine to second-order derivatives. A similar algorithm has been derived for sound fields [11]. Although improved and more accurate FDTD algorithms have been developed for aeroacoustic applications (e.g., based on approximations of higher-order, unstaggered meshes, upwind schemes[12,13]), Yee's staggered algorithm remains an economical and robust way to carry out the FDTD algorithm [14], giving a compromise between accuracy and efficiency. This scheme has been widely used in fields such as room acoustic applications [2,3,15,16], musical sound synthesis [17–19], and outdoor sound propagation [20–22].

In the staggered FDTD in acoustics, the scalar pressure and the three components of the particle velocity are distributed around an acoustic Yee unit cell [11], which is similar to the original electromagnetic FDTD cell, but based on conservation of mass and momentum (see Fig. 1),

$$\frac{\partial p(\mathbf{r},t)}{\partial t} = -\rho_0 c^2 \nabla \cdot \mathbf{u}(\mathbf{r},t), \tag{1}$$

$$\rho_0 \frac{\partial \mathbf{u}(\mathbf{r},t)}{\partial t} = -\nabla p(\mathbf{r},t), \tag{2}$$

where p is the sound pressure, $\mathbf{u}$ is the acoustic particle velocity, $\mathbf{r}$ is the position, t is the time, c is the speed of sound, and ρ_0 is the density of air.

The FDTD method studied in this paper uses a second-order central finite-difference approach to the derivatives [23]. In Cartesian coordinates Eqs. (1) and (2) become (in 2D) the following system of discretized equations:

$$p(i,j,n+1) = p(i,j,n) - \frac{\rho_0 c^2 \Delta t}{\Delta x}[u_x(i+\tfrac{1}{2},j,n+\tfrac{1}{2}) - u_x(i-\tfrac{1}{2},j,n+\tfrac{1}{2})] - \frac{\rho_0 c^2 \Delta t}{\Delta y}[u_y(i,j+\tfrac{1}{2},n+\tfrac{1}{2}) - u_y(i,j-\tfrac{1}{2},n+\tfrac{1}{2})], \tag{3}$$

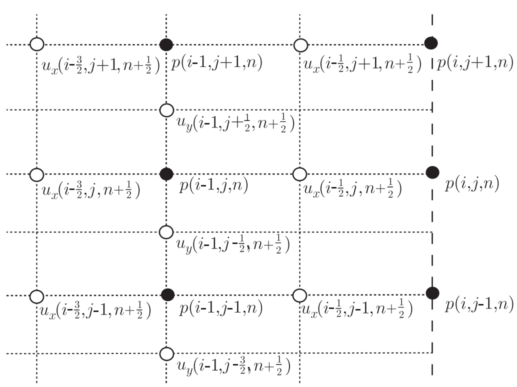

Fig. 1. Acoustic Yee unit cell in FDTD algorithm.