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The transition conditions in the dynamics of elastically restrained beams

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Abstract

This paper deals with the free transverse vibration of a non-homogeneous tapered beam subjected to general axial forces, with arbitrarily located internal hinge and elastics supports, and ends elastically restrained against rotation and translation. A rigorous and complete development is presented. First, a brief description of several papers previously published is included. Second, the Hamilton principle is rigorously stated by defining the domain D of the action integral and the space D_a of admissible directions. The differential equations, boundary conditions, and particularly the transitions conditions, are obtained. Third, the transition conditions are analysed for several sets of restraints conditions. Fourth, the existence and uniqueness of the weak solutions of the boundary value problem and the eigenvalue problem which, respectively, govern the statical and dynamical behaviour of the mentioned beam is treated. Finally, the method of separation of variables is used for the determination of the exact frequencies and mode shapes and a modern application of the Ritz method to obtain approximate eigenvalues. In order to obtain an indication of the accuracy of the developed mathematical model, some cases available in the literature have been considered. New results are presented for different boundary conditions and restraint conditions in the internal hinge.

1. Introduction

The calculus of variations is the oldest and most important root of functional analysis. Lagrange invented the ‘‘operator’’ δ and with its application a δ-calculus which was viewed as a kind of ‘‘higher’’ infinitesimal calculus. This discipline has attracted the attention of numerous eminent mathematicians, who made important contributions to its development. In the last decades the interest in application of the techniques of the calculus of variations has increased noticeably. This is partly due to the demands of the technology and the availability of powerful computers.

Variational principles have always played an important role in theoretical mechanics. In 1717 Johann Bernoulli presented the principle of virtual work and in 1835 Hamilton's principle emerged. Particularly, this

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Nomenclature

$A_i(x)$	cross-sectional area of the ith span, $i = 1,2$
$D(F)$	domain of functional F
$D_a(F)$	space of admissible directions
$D_i(x) = E_i(x)I_i(x)$	flexural rigidity of the ith span
$F(u)$	energy functional
$f(x)$	distributed axial force
K_{ri}, K_{ti}, $i = 1,2$	dimensionless rotational and translational parameters
K_{rc}, K_{tc}	dimensionless rotational and translational parameters
K_{r12}	dimensionless rotational parameter
l	length of the beam
$m_i(x) = \rho_i(x)A_i(x)$	mass per unit length of the ith span
$\mathbb{N}$	the set of natural numbers
$\mathbb{R}^n$	n-dimensional Euclidean space
$\mathbb{R}$	the set of real numbers
r_1, r_2	rotational stiffness at the left and right ends, respectively
r_c	rotational stiffness at the point $x = c$
r_{12}	rotational stiffness at the internal hinge
S_i	dimensionless axial force
$T(x)$	axial load at abscissa x
T_b	kinetic energy
t	time
t_1, t_2	translational stiffness at the left and right ends, respectively
t_c	translational stiffness at the point $x = c$
U	strain energy
x	abscissa
$\bar{x}$	dimensionless abscissa
$\lambda_{1,i} = \sqrt[4]{\frac{m_1}{D_1}\omega_i^2}\,l$	dimensionless natural frequency parameter
$\delta F(u; v)$	variation of functional F
ω	radian frequency
$\rho_i(x)$	mass density of the ith span

last principle provides a straightforward method for determining equations of motion and boundary conditions of mechanical systems. Substantial literature has been devoted to the theory and applications of the calculus of variations. For instance, the excellent books [1–3] present clear and rigorous treatments of the theoretical aspects of the mentioned discipline. Several classical textbooks, [4–8] present formulations, by means of variational techniques, of boundary value and eigenvalue problems in the statics and dynamics of mechanical systems.

On the other hand, the study of vibration problems of beams with several complicating effects has received considerable treatment. It is not possible to give a detailed account because of the great amount of information, nevertheless some references will be cited. Excellent handbooks have appeared in the literature giving frequency tables and mode-shape expressions [9,10]. Several investigators have studied the influence of rotational and/or translational restraints at the ends of vibrating beams. A number of previous papers have been published on uniform beams with elastically restrained ends [11–18]. Transverse vibrations of beams of non-uniform cross sections have also been extensively investigated [19–27].

Also, the study on vibration of beams with intermediate elastic restraints has been performed by several researchers. Rutemberg [28] presented eigenfrequencies for a uniform cantilever beam with a rotational restraint at an intermediate position. Lau [29] extended Rutemberg's results including an additional spring to against translation. Rao [30] analysed the frequencies of a clamped–clamped uniform beam with intermediate elastic support. De Rosa et al. [31] studied the free vibrations of stepped beams with intermediate elastic supports. Arenas and Grossi [32] presented exact and approximate frequencies of a uniform beam, with one end spring-hinged and a rotational restraint in a variable position. Grossi and Albarracín [33] determined the exact eigenfrequencies of a uniform beam with intermediate elastic constraints. Wang [34] determined the minimum stiffness of an internal elastic support to maximize the fundamental frequency of a vibrating beam.

A review of the literature further reveals that there is only a limited amount of information for the vibration of beams with internal hinges. Ewing and Mirsafian [35] analysed the forced vibrations of two beams joined with a nonlinear rotational joint. Wang and Wang [36] studied the fundamental frequency of a beam with an internal hinge and subjected to an axial force. Chang et al. [37] investigated the dynamic response of a beam with an internal hinge, subjected to a random moving oscillator. The aim of the present paper is to investigate the natural frequencies and mode shapes of a beam with several complicating effects, intending the development within each section to be rigorous and complete.