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Vibration analysis of multiple-stepped beams with the composite element model

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Abstract

A new approach to analyze the free and forced vibrations of beams with multiple cross-section steps is proposed using a composite element method in this paper. The results are compared to receptance function method and classical Rayleigh–Ritz method and finite element results. The natural frequencies obtained from the composite element method are found to be in close agreement with other methods mentioned above and finite element method. The forced vibration responses of the stepped beam are also calculated from the composite element method and compared with those obtained from the finite element method. Time histories from both methods are found to match very well. This indicates the correctness of the proposed method for vibration analyses of the stepped beam. The proposed method can be extended easily to deal with beams consisting of an arbitrary number of nonuniform segments.

1. Introduction

Beams are fundamental models for the structural elements of many engineering applications and have been studied extensively. There are many examples of structures that may be modeled with beam-like elements, for instance, long span bridges [1], tall buildings [2], and robot arms [3].

The vibration of Euler–Bernoulli beams with one step change in cross-section has been well studied. Jang and Bert [4,5] derived the frequency equations for combinations of classical end supports as fourth-order determinants equated to zero. Taleb and Suppiger [6] derived the frequency equation of a stepped beam with simple support using the Cauchy iteration method. Balasubramanian and Subramanian [7,8] investigated the performance of a four-degree-of-freedom per node element in the vibration analysis of a stepped cantilever. Popplewell and Daqing Chang [9] used the "force mode functions" to improve the global Rayleigh–Ritz method of convergence by introducing discontinuities into the second and third derivatives of the assumed deflection. Krishnan et al. [10] discussed the difficulties of a finite difference analysis for a pinned–pinned stepped beam. De Rosa [11] studied the vibration of a stepped beam with elastic end supports. Recently,

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Koplow et al. [12] presented closed form solutions for the dynamic response of Euler–Bernoulli beams with step changes in cross-section.

There are also some works on the vibration of beams with more than one step change in cross-section. Gorman [13] analyzed exactly the vibrating properties of one-step beams with several boundary condition combinations and symmetrical two-step beams on identical end supports. Bapat and Bapat [14] proposed the transfer matrix approach for beams with n-steps but provided no numerical results. Ju et al. [15] used a first-order shear deformation theory and the corresponding finite element formulation to study the free vibrations of clamped–clamped and cantilevered beams with one or two step changes in cross-section. Lee and Bergman [16] used the dynamic flexibility method to derive the frequency equation of a beam with n-step changes in cross-section. Naguleswaran [17] studied a beam with up to three steps with 45 combinations of classical and elastic end supports using the frequency equation method of Jang and Bert [4,5]. Jaworski and Dowell [18] carried out a study for the free vibration of a cantilevered beam with multiple steps and compared the results of several theoretical methods with experiment.

In this paper, a new method is presented to analyze the free and forced vibrations of beams with either a single step change or multiple step changes using the composite element method (CEM) [19,20]. The correctness and accuracy of the proposed method are verified by some examples in the existing literatures. The principal advantage of the proposed method is that it does not need to partition the stepped beam into uniform beam segments between any two successive discontinuity points and the whole beam can be treated as a uniform beam. Moreover, the presented work can easily be extended to beams with an arbitrary number of nonuniform segments.

2. Theory

2.1. Brief introduction to the CEM

The composite element is a relatively new tool for finite element modeling. This method is basically a combination of the conventional finite element method (FEM) and the highly precise classical theory (CT). In the CEM, the displacement field is written as the sum of the finite element displacement and the shape functions from the CT. The displacement field of the CEM can be expressed as

$$u_{\mathrm{CEM}}(x,t) = u_{\mathrm{FEM}}(x,t) + u_{\mathrm{CT}}(x,t), \tag{1}$$

where $u_{\mathrm{FEM}}(x,t)$ and $u_{\mathrm{CT}}(x,t)$ are the individual displacement fields from the FEM and CT, respectively.

Taking a planar beam element as an example, the first term of the CEM displacement field can be expressed as the product of the shape function vector $N(x)$ and the nodal displacement vector q

$$u_{\mathrm{FEM}}(x,t) = N(x)q(t), \tag{2}$$

where $q(t) = [v_1(t), \theta_1(t), v_2(t), \theta_2(t)]^{\mathrm{T}}$ and 'v' and 'θ' represent the transverse and rotational displacements, respectively, and

$$\begin{aligned} N(x) &= \left[1 - 3\left(\frac{x}{L}\right)^2 + 2\left(\frac{x}{L}\right)^3, \frac{x}{L} - 2\left(\frac{x}{L}\right)^2 + \left(\frac{x}{L}\right)^3, 3\left(\frac{x}{L}\right)^2 - 2\left(\frac{x}{L}\right)^3, \left(\frac{x}{L}\right)^3 - \left(\frac{x}{L}\right)^2\right], \\ &= [N_1(x), N_2(x), N_3(x), N_4(x)]. \end{aligned} \tag{3}$$

The second term $u_{\mathrm{CT}}(x,t)$ is obtained by the multiplication of the analytical mode shapes with a vector of N coefficients c (also called the c degrees-of-freedom or c-coordinates)

$$u_{\mathrm{CT}}(x,t) = \sum_{i=1}^{N} \varphi_i(x) c_i(t), \tag{4}$$

where φ_i ($i = 1,2,\ldots,N$) is the analytical shape function of the beam. Different analytical shape functions are used according to the boundary conditions of the beam.

Like the FEM, the CEM can be refined using the h-refinement technique by increasing the number of finite elements. Moreover, it can also be refined through the c-refinement method, by increasing the number of