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The influence of disk's flexibility on coupling vibration of shaft–disk–blades systems

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Abstract

The coupling vibrations among shaft-torsion, disk-transverse and blade-bending in a shaft–disk–blades unit are investigated. The equations of motion for the shaft–disk–blades unit are first derived from the energy approach in conjunction with the assumed modes method. The effects of disk flexibility, blade's stagger angle and rotational speed upon the natural frequencies and mode shapes are particularly studied. Previous studies have shown that there were four types of coupling modes, the shaft–blade (SB), the shaft–disk–blades (SDBs), the disk–blades (DB) and the blade–blade (BB) in such a unit. The present research focuses on the influence of disk flexibility on the coupling behavior and discovers that disk's flexibility strongly affects the modes bifurcation and the transition of modes. At slightly flexible disk, the BB modes bifurcate into BB and DB modes. As disk goes further flexible, SB modes shift into SDB modes. If it goes furthermore, additional disk-predominating modes are generated and DB modes appear before the SDB mode. Examination of stagger angle β proves that at two extreme cases; at $\beta = 0^\circ$ the shaft and blades coupled but not the disk, and at $\beta = 90^\circ$ the disk and blades coupled but not the shaft. In between, coupling exists among three components. Increasing β may increase or decrease SB modes, depending on which, the disk or shaft's first mode, is more rigid. The natural frequencies of DB modes usually decrease with the increase of β. Rotation effects show that bifurcation, veering and merging phenomena occur due to disk flexibility. Disk flexibility is also observed to induce more critical speeds in the SDBs systems.

1. Introduction

A shaft–disk–blades unit, as shown in Fig. 1, has been widely used in rotating machinery. Applications range from turbine generators, turbine engines, to rotor compressors, etc. Dynamic couplings among shaft, disk and blades could became significant if the disk is flexible and the blade's stagger angle varies. Many researches concerning individual vibrations of shaft, disk and blades have been published worldwide. However, coupling vibrations among shaft torsion, disk transverse and blade bending is rarely addressed. The present research focuses on this issue and explores what extent the coupling relies on disk flexibility and blade's stagger angle.

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Nomenclature

V_{ki}	the ith mode shape of the kth blade
$\hat{v}_{b_k}$	kth blade displacements with respect to the Y_3 axis
v_b, w_b	blade displacements with respect to the Y_2 and Z_2 axes
w_d	disk transverse displacement with respect to the Z_1 axis
β	stagger angle
ϕ	shaft–disk torsional displacement relative to rotation frame
Φ_i, W_i	the ith mode shape of the shaft–disk and disk
$\boldsymbol{q}$, $\boldsymbol{\eta}$, $\boldsymbol{\zeta}$, $\boldsymbol{\xi}$	vectors consisting of generalized coordinates
Ω	rotational speed of shaft speed
Ω^*	dimensionless rotational speed ($\Omega^* = \Omega/\omega_1^b$)
ω_1^b	the first nature frequency of single cantilever blade
ω	natural frequency
ω^*	dimensionless natural frequency ($\omega^* = \omega/\omega_1^b$)

Subscripts

$(\ )_s$	shaft
$(\ )_d$	disk
$(\ )_b$	blade
$(\ )_{ki}$	ith term of the kth blade

Superscripts

$(\ )^n$	n-nodal diameter of disk

Dynamic characteristics of disk flexibility have been studied for years. Lamb and Southwell [1] derived the frequency equations and mode shapes for a circular spinning disk. Southwell [2] extended that investigation to an annular plate clamped inside and free outside in a following paper. Dopkin and Shoup [3] discussed the influence of disks' flexibility on natural frequencies using transfer matrix method. Shen and Ku [4] studied natural frequencies and modes shapes of multiple elastic disks on a rigid shaft through the use of Lagrangian mechanics. The vibration characteristics of the shaft–disk [5–8] and disk–blade [9–12] were studied as well but not the coupling effects among components were discussed. Chivens and Nelson [13] investigated the influence of disk flexibility on bending natural frequencies and critical speeds of a flexible shaft–disk system using the Laplace transform method. They concluded that critical speeds were insignificant to disk flexibility, but natural frequencies did. Shahab and Thomas [14] discussed the coupling effects of shaft and disk flexibility on a shaft-multiple disks system using the finite element method. Their investigation revealed that disk rigidity imposed less effect than shaft on system modes. Shen [15] used the assumed modes method to study the natural frequencies and mode shapes of a shaft-multiple disks system on forced vibration. Wu and Flowers [16] investigated the coupling effects of shaft and disk using the transfer matrix method. Jia et al. [17] used the assumed mode method to investigate the longitudinal coupled vibration of a flexible shaft with multiple flexible disks. Lee and Chun [18] investigated the effects of multiple flexible disks on the vibration modes of a flexible shaft–disk system via using assumed modes method. The frequency bifurcations of shaft and disk coupling modes occurred due to disk flexibility. Omprakash and Ramamurti [19] analyzed the natural frequencies of rotating disk–blade system by a combined cycle symmetry and Rayleigh–Ritz method. Omprakash and Ramamurti [20] discussed the influence of stagger and pretwist angle on the coupling vibration characteristics of disk–blade system by the finite element method. They investigated the influence of disk flexibility and found that disk–blades coupling frequencies approached the blade's frequencies as the number of disk's nodal diameters increased. When the stagger angle was zero, the disk and blades were completely uncoupled. As to the dynamic characteristics of a shaft–disk–blades unit, there were limited investigations in the existing literature. Khader and Loewy [21] discussed the forced response of a rotating shaft–disk–blades system by modal analysis method and focused on the influence of Coriolis force. Sakata et al. [22] investigated the influence of gyroscopic moment of the shaft–disk–blades system by the finite element method. Chun and Lee [23] used the assumed mode method to investigate the influence of the forces and torques, resulted from changes of blade's stagger and pretwist angles, on the dynamic coupling of a flexible shaft–disk–blades unit. This investigation had arrived at some conclusions: the pretwist angle had far less influence on the system dynamics than the stagger angle, and the torque and coupling effect on the shaft