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Modeling wave propagation in damped waveguides of arbitrary cross-section

Ivan Bartoli[a,b], Alessandro Marzani[a,b], Francesco Lanza di Scalea[a,*], Erasmo Viola[b]

[a]*NDE & Structural Health Monitoring Laboratory, Department of Structural Engineering, University of California, San Diego, 9500 Gilman Drive, La Jolla, CA 92093-0085, USA*

[b]*Dipartimento di Ingegneria delle Strutture, dei Trasporti, delle Acque, del Rilevamento, del Territorio (DISTART), Universita' degli Studi di Bologna, Via Risorgimento 2, 40136 Bologna, Italia*

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Abstract

This paper deals with a semi-analytical finite element (SAFE) method for modeling wave propagation in waveguides of arbitrary cross-section. The method simply requires the finite element discretization of the cross-section of the waveguide, and assumes harmonic motion along the wave propagation direction. The general SAFE technique is extended to account for viscoelastic material damping by allowing for complex stiffness matrices for the material. The dispersive solutions are obtained in terms of phase velocity, group velocity (for undamped media), energy velocity (for damped media), attenuation, and cross-sectional mode shapes. Knowledge of these properties is important in any structural health monitoring attempt that uses ultrasonic guided waves. The proposed SAFE formulation is applied to several examples, including anisotropic viscoelastic layered plates, composite-to-composite adhesive joints and railroad tracks.

1. Introduction

Guided ultrasonic waves provide a highly efficient method for the non-destructive evaluation (NDE) and the structural health monitoring (SHM) of solids with finite dimensions. Compared to ultrasonic bulk waves, guided waves provide larger monitoring ranges and the complete coverage of the waveguide cross-section. Compared to global vibrations, guided waves provide increased sensitivity to smaller defects due to the larger frequencies. These advantages can be fully exploited only once the complexities of guided wave propagation are unveiled and managed for the given test structure. These complexities include the existence of multiple modes, the frequency-dependent velocities (dispersion), and the frequency-dependent attenuation. For example, the knowledge of the wave velocity is important for mode identification. Similarly, the knowledge of those mode–frequency combinations propagating with minimum attenuation losses helps maximizing the inspection coverage.

Semi-analytical finite element (SAFE) methods, also referred to in the literature as spectral or waveguide finite element methods, have emerged for modeling the guided wave propagation numerically as an alternative

*Corresponding author. Tel.: +1 858 822 1458; fax: +1 858 534 6373.
E-mail address: flanza@ucsd.edu (F. Lanza di Scalea).

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to the "exact" methods based on the superposition of bulk waves—SPBW, that include the popular matrix-based methods [1]. Motivations for the numerical methods include the necessity for modeling a large number of layers such as composite laminates and that of modeling waveguides with arbitrary cross-section for which exact solutions do not generally exist. In addition, when complex wavenumbers are part of the solution such as in the case of leaky and/or damped waveguides, the exact SPBW methods require iterative bi-dimensional root-searching algorithms that may miss some of the solutions [1].

The general SAFE approach for extracting dispersive solutions uses a finite element discretization of the cross-section of the waveguide alone. The displacements along the wave propagation direction are conveniently described in an analytical fashion as harmonic exponential functions. Thus only a bi-dimensional discretization of the cross-section is needed, with considerable computational savings compared to a 3-D discretization of the entire waveguide. The SAFE solutions are obtained in a stable manner from an eigenvalue problem, and thus do not require the root-searching algorithms used in SPBW approaches. In addition, since polynomial approximation of the displacement field along the waveguide is avoided, the method is applicable to predicting waves with very short wavelengths, where a traditional 3-D approximation may fail.

A SAFE method for waveguides of arbitrary cross-section was demonstrated for the first time in 1973 [2,3]. In these works dispersive solutions were obtained for the propagative modes only (i.e. real wavenumbers only). The same technique was used a decade later [4] to calculate both propagative modes and nonpropagative, evanescent modes (complex wavenumbers) for anisotropic cylinders. While the evanescent modes do not transport any energy along the structure, they are important from a theoretical viewpoint to satisfy the boundary conditions. More recently, SAFE methods confined to obtaining the propagative solutions were applied to thin-walled waveguides [5], railroad tracks [6] and wedges [7]. An approximation of the method in Refs. [5,6] was also implemented in a standard finite element package by imposing a cyclic axial symmetry condition [8]. An extension can be found in Ref. [9] which examined waveguides immersed in water. Other versions of the general SAFE method, again for the propagative modes, were applied to nonhomogeneous anisotropic beams [10], rods and rails [11]. Both propagative and evanescent modes in twisted waveguides were studied by SAFE methods in Ref. [12]. Reflection phenomena from the end of a waveguide were studied in Ref. [13]. Modes in built-up thin-walled structures, including a channel beam and a plate in a wind tunnel, were examined in Ref. [14]. In this work an interesting formulation was presented for obtaining the group velocity values from the individual solutions of the SAFE eigenproblem. This is advantageous compared to the incremental calculations that are required in the conventional derivation of the group velocity defined as $c_g = \partial\omega/\partial\xi$ (ω is the frequency and ξ is the wavenumber). Laminated composite waveguides were studied by SAFE methods for the first time in Ref. [15] and, subsequently, in Refs. [16,17] for laminated plates of both finite and infinite widths.

The focus of previous SAFE works was obtaining propagative and evanescent modes in undamped waveguides. A need exists to extend this technique to account for material damping. One very recent work [18] demonstrates a SAFE application to damped, viscoelastic composite laminates. In this reference a damping loss factor was estimated indirectly from the power dissipated by the wave. However, the formulation in Ref. [18] still does not allow for the calculation of the true wave attenuation since the governing stiffness matrix was assumed real. In Ref. [19], another damping loss factor was considered in a complex formulation for the material's Young's modulus. The focus of this reference was on global dynamic behavior of plate systems rather than ultrasonic guided waves.

The present study extends the SAFE method for modeling dispersive solutions in waveguides of arbitrary cross-sections by accounting for material damping. This extension is particularly relevant for NDE/SHM applications on high-loss materials such as viscoelastic fiber-reinforced polymer composites. When accounting for damping, the exact energy velocity, rather than the conventional group velocity, is calculated along with the frequency-dependent attenuation of the modes. Various examples are shown, including isotropic plates, composite laminates, composite-to-composite adhesive joints and railroad tracks.

2. Viscoelastic models for wave propagation

This section reviews the linear viscoelastic models that were used in the SAFE formulation proposed in the present work. As well known, for time harmonic motion $e^{-i\omega t}$, linear viscoelasticity can be modeled by