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# Finite cylinder vibrations with different end boundary conditions

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**Abstract**

Utilizing the infinite circular cylinders solution based on the technique of variables separation, a general solution is developed to analyze the vibration of finite circular cylinders. The vibration of finite circular cylinders with different end boundary conditions as well as the curved panels can be analyzed by the semi-analytical method developed in the present study. In the present paper two different boundary conditions are considered, namely the free-end and fixed-end hollow cylinders. Convergence and precision of the method are determined to calculate the natural frequencies of various geometrical configurations. It is shown that the results obtained from the present semi-analytical method are in good agreement with those obtained using the previously developed methods. Generality, high accuracy and good convergence with a small sized of coefficient matrix are the merits of the present method.


## 1. Introduction

Finite length hollow cylinders are indispensable in many industries such as marine structures, necessitating the thorough comprehension of their vibration with different boundary conditions. These understanding may be used to analyze the sound transmission through the single and multilayered finite cylinders. The earliest investigation concerning the vibration of cylinders was performed by Pochhamer [1] and Chree [2]. The Pochhamer–Chree solution was developed for an infinitely long solid cylinder. Greenspon [3], Gazis [4] and Armenakas [5] studied the vibration of infinitely long traction free hollow cylinders using linear three dimensional (3D) theory of elasticity. McNevin et al. [6] developed a three-mode theory for axisymmetric vibrations of rods and hollow cylinders. Gladwell and Tahbildar [7] investigated axisymmetric vibrations of cylinders using the finite-element method. The vibration of free finite length circular cylinders using the finite-element method was analyzed by Gladwell and Vijay [8]. Hutchinson [9,10] developed a semi-analytical highly accurate method to solve the vibrations of finite length rods and solid cylinders on the basis of linear 3D elasticity. Hutchinson and El-Azhari [11] investigated the vibrations of free hollow finite length circular

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**Nomenclature**

| | |
|---|---|
| $a$, $b$ | inner and outer radii of the cylinder |
| $A_k$, $B_k$, $F_k$, $G_k$, $\bar{F}_k$, $\bar{G}_k$ | constants |
| $C_1$, $C_2$ | propagation velocity of dilatational and distortional waves |
| $H$ | ratio of the thickness with respect to the mean radius |
| $\mathbf{H}$ | vector potential functions |
| $I_v$, $K_v$ | modified Bessel functions (order $v$) |
| $J_v$, $Y_v$ | bessel and Neumann functions (order $v$) |
| $l$ | half length of the cylinder |
| $L$ | ratio of the length over the mean radius |
| $n$ | circumferential wave number |
| $N_1$, $N_2$ | number of truncated series terms in $z$ and $r$ directions |
| $r$, $\theta$, $z$ | cylindrical coordinates |
| $t$ | time |
| $\mathbf{u}$ | displacement vector |
| $\lambda$, $\mu$ | lame constants |
| $\rho$ | density |
| $\boldsymbol{\sigma}$ | stress field |
| $\varphi$ | scalar potential functions |
| $\omega$ | circular frequency |
| $\Omega = \omega b/C_2$ | non-dimensional natural frequency |
| $\nabla^2$ | three-dimensional Laplacian operator |

cylinders using the method which had been previously reported by Hutchinson. In Hutchinson's heretofore studies different forms of solutions, by combining some fundamental solution forms are suggested for the above mentioned cases. Singal and Williams [12] investigated the vibrations of thick hollow cylinders using the energy method based on the 3D theory of elasticity. Leissa and So [13,14] studied the vibrations of free and cantilevered solid cylinders using simple algebraic polynomials in the Ritz method. Liew et al. [15–17] studied the free vibrations of solid and hollow cylinders with different end boundary conditions using 3D energy displacement-based expressions. The convergence of the method and parametric investigations were performed for different boundary conditions and cross-sections of hollow cylinders. Some studies have also been performed on the vibrations of the cylinders that include the classification of natural frequencies or mode shapes such as the one presented by Wang and Williams [18] using the finite element method. Modified methods are also used to obtain more accurate and better convergence of the results, for example Zhou et al. [19] studied 3D vibrations of the solid and hollow cylinders using the Ritz method and Chebyshev polynomials.

In this paper a general semi-analytical solution using the technique of variables separation on the basis of linear 3D theory of elasticity is developed which covers different cases of finite length cylinders such as rods, solid cylinders, hollow cylinders and curved panels with various boundary conditions. In this method some of the boundary conditions need to be approximately satisfied using orthogonalization technique while the others to be exact. Comparing with the previously developed series solutions, high accuracy and good convergence with a small sized of coefficient matrix are achieved in the eigenvalues estimation using the present method.

## 2. Formulation

The geometry of a typical hollow circular cylinder is shown in Fig. 1. An orthogonal cylindrical coordinate ($r$, $\theta$, $z$) system is considered as shown in this figure. The corresponding components of the displacement vector $\mathbf{u}$ at a point are $u_r$, $u_\theta$ and $u_z$ in the $r$, $\theta$, and $z$ directions, respectively. The displacement equations governing the motion of an isotropic media are

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = \mu \nabla^2 \mathbf{u} + (\lambda + \mu)\nabla(\nabla.\mathbf{u}), \tag{1}$$

where $\rho$ is the density, $\lambda$ and $\mu$ are the Lame constants, and $\nabla^2$ is the 3D Laplacian operator. The most general solution of Eq. (1) may be obtained using Helmholtz decomposition as follows:

$$\mathbf{u} = \nabla\varphi + \nabla \times \mathbf{H} \tag{2}$$

with the condition of

$$\nabla.\mathbf{H} = F(r, \theta, z, t), \tag{3}$$