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On the use of Pade approximants in the estimation of eigenfrequencies and damping ratios of a vibrating system

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Abstract

An approach to estimate eigenfrequencies and damping ratios of a vibrating system, in time domain from output data only, is studied. This approach is based on the interpretation of histograms obtained from the poles of Padé approximants. Using properties of the asymptotic location of poles of Padé approximants to rational functions, different subsets of eigenfrequencies and damping ratios are obtained and their histograms plotted. Numerical and experimental examples are presented.

1. Introduction

Modal parameter identification is used to identify those parameters of the model which describe the dynamic properties of a vibrating system. Classical modal parameter extractions usually require measurements of both: the input force and the resulting response in laboratory conditions. However, for some practical reasons, when operational structures are subjected to random and unmeasured forces such as wind, waves, traffic, shocks, or aerodynamics, modal parameters must be extracted from response-only data. The problem of output-only modal analysis has gained considerable attention in recent years and several different approaches to estimate modal parameters from output-only data have been proposed. They include peak-picking from power spectral density functions, autoregressive moving average models, subspace techniques and wavelet transform [1–10]. Output-only methods offer undeniable advantages: they can be applied without interrupting the structure regular service during the experimental tests; they require no special excitation equipment; it is not necessary to know or measure the excitation. The modal parameters play a relevant role in structural monitoring and inspection. In fact, changes in modal parameters may reflect changes in local mechanical properties due to damaging phenomena underway.

In this paper, a statistically procedure for the estimation of natural frequencies and damping parameters of structural systems under white-noise input is shown. The procedure is based on the computation of Padé

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approximants to the Z-transform of a noisy data sequence [11–14]. Some theoretical properties of poles of Padé approximants to noisy rational functions are then developed.

The paper is organized as follows. First, a model of a vibrating system and its state space representation are given. The poles of the system are related to covariance matrices and modal parameters are obtained from these poles. Since the order of the state space model, or the number of modes, is unknown, we increase this order. Then, the number of Padé approximants increases, and spurious poles appear. We propose statistical methods to eliminate such spurious modes and to determine the true modes. A numerical example and an experimental test are then presented and the paper is briefly concluded.

2. Modelling a vibrating structure

We consider a structure excited by an unknown white-noise input. Our objective is to determine the modal parameters, from the time response delivered by the output of accelerometers in contact with the structure.

For an n'-degree of freedom vibratory system, the equation of motion can be expressed as

$$M_0\ddot{\xi}(t) + C_0\dot{\xi}(t) + K_0\xi(t) = \eta(t), \tag{1}$$

where M_0, C_0 and K_0 are the system mass, damping and stiffness matrices $(n' \times n')$ respectively; $\ddot{\xi}(t)$, $\dot{\xi}(t)$ and $\xi(t)$ are $(n' \times 1)$ vectors of acceleration, velocity and displacement and $\eta(t)$ the $(n' \times 1)$ unmeasured excitation vector, which is a white-noise sequence. A state-space model can be formed in lieu of the model given by Eq. (1) as

$$\dot{x}(t) = \tilde{A}x(t) + \tilde{B}\eta(t), \tag{2}$$

where $x(t)$ is the $n = 2n'$ dimensional state vector:

$$x(t) = \begin{bmatrix} \xi(t) \\ \dot{\xi}(t) \end{bmatrix}$$

and $\tilde{A}$, $\tilde{B}$ are given by

$$\tilde{A} = \begin{bmatrix} 0 & I \\ -M_0^{-1}K_0 & -M_0^{-1}C_0 \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} 0 \\ M_0^{-1} \end{bmatrix}.$$

The response of the dynamic system is measured by the r output quantities in the output $y(t)$, using accelerometers. An $(r \times 1)$ vector output equation, called the observation equation, can be written as [2] $\ddot{\xi}$

$$y(t) = H_a\ddot{\xi}(t) = H_aM_0^{-1}[-K_0\xi(t) - C_0\dot{\xi}(t) + \eta(t)] \quad \text{or} \quad y(t) = Cx(t) + D\eta(t), \tag{3}$$

where H_a is the output influence matrix $(r \times n')$ for acceleration. This matrix specifies which points of the system are observed from accelerometers. C is the $(r \times n')$ output influence matrix for the state vector $x(t)$ given by $C = H_aM_0^{-1}[-K_0 - C_0]$ and D is an $(r \times n')$ direct transmission matrix given by $D = H_aM_0^{-1}$. Eqs. (2) and (3) constitute the continuous time state space model of a dynamical system. After sampling with constant period Δt and transformation of the $2n'$ first-order differential equations (2) and (3) into a discrete time equation, we obtain the following discrete time state-space model, where a process noise due to disturbances and modelling inaccuracies is added [2,4]

$$x_{k+1} = Ax_k + w_k, \tag{4}$$

where x_k represents the discrete unobserved state vector of dimension $n = 2n'$; $A = e^{\tilde{A}\Delta t}$ is the $(n \times n)$ discrete time transition matrix; w_k is given by $w_k = \int_0^{\Delta t} e^{\tilde{A}s}\tilde{B}\eta(t-s)\,ds + w'_k$ with w'_k the process noise. The discrete time observation equation with measurement noise is given by

$$y_k = Cx_k + v_k, \tag{5}$$

where $v_k = D\eta_k + v'_k$ and v'_k is the measurement noise.