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Dynamic response of a two-level catenary to a moving load

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Abstract

An analytical method is proposed for calculating the steady-state response of a two-level catenary to a uniformly moving pantograph. The model for the catenary is composed of two strings (the contact and carrying cables) connected by lumped mass-spring-dashpot elements (hangers), which are positioned equidistantly along the strings. The upper string (carrying cable) is fixed at periodically spaced points. This model is capable of describing a coupled wave dynamics of both the carrying cable and the contact cable of the catenary. The pantograph is modelled by a point load, which moves uniformly along the contact cable. Using the proposed method, the steady-state deflection of the contact cable is analyzed thoroughly. Additionally, the contact force between the hangers and the contact cable is studied, which is important for estimation of the fatigue life of the hangers. Two simplified models of the two-level catenary are introduced and studied. The first model assumes that the carrying cable is infinitely stiff, whereas the second model disregards the discrete character of the hangers. Predictions of these simplified models are compared to those of the original model.

1. Introduction

Overhead catenary systems for high-speed trains require a relatively high tension of both the carrying cable (carrier) and the contact cable. This is needed to prevent the train velocity from getting close to the wave speed of flexural waves in the catenary. It was measured, however, that a higher tension leads to a higher contact loss ratio of the pantograph. To avoid this effect, which leads to a lower efficiency of the current collection, it was proposed to replace conventional droppers (simple cables, on which the contact wire is suspended) by more sophisticated rubber damping hangers or friction damping hangers [1]. These hangers have certain stiffness, viscosity and mass, which can be tuned to minimize wave reflection from the hangers. According to Ref. [1], such tuning allows to reduce the contact loss ratio.

Introduction of the hangers, which are much stiffer in compression than the conventional droppers (the latter have nearly zero stiffness in compression), leads to much more intense an interaction between the cables of the catenary under the pantograph, whose influence is mainly compressive. To account for this interaction, coupled vibrations of the cables should be considered.

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In this paper, an analytical method is proposed for obtaining the steady-state response of a two-level catenary to a uniformly moving pantograph. The model for the catenary is composed of two strings connected by lumped mass-spring-dashpot elements (hangers). This model is capable of describing a coupled wave dynamics of both the carrying cable and the contact cable. The pantograph is modelled by a point load of a constant or a harmonically varying with time magnitude. Adopting this model of the pantograph, it is implicitly assumed that regardless of the pantograph velocity, the coupled vibrations of the pantograph–catenary system are stable and the steady-state vibrations of this system exist. As shown by Metrikine and Verichev [2], the stable interaction can always be achieved by a proper choice of the effective mass, stiffness and viscosity of the pantograph (the viscosity is the most influential parameter in this case). Note that in this paper the existence of the stable interaction is assumed but the pantograph parameters, which would ensure this, are not determined.

The emphasis of this study is placed on the deflection of the contact cable and its dependence on the load speed. Additionally, the contact force between the hangers and the contact cable is discussed. To reveal significance of the coupling between the two cables, which takes place by means of the lumped hangers, the response of the two-level catenary is compared to that of two simplified models. In the first simplified model the carrying cable is assumed infinitely stiff but effective parameters of the droppers are introduced to account for the stiffness and damping properties of this cable. In the second model, the simplification is concerned with the hangers, which are 'continualized', forming a continuous and homogeneous visco-elastic connection between the two cables.

All three models, which are considered in this paper, belong to a class of periodically inhomogeneous, continuous elastic systems. Such systems, being excited by a uniformly moving load, can respond in a resonance manner at several load velocities. This effect has been studied in the past by a number of researchers employing different but closely related methods. Mead and Jezequel based their approach on the Fourier-series techniques [3–6]. Bogacz, Krzyzinski and Popp applied the Flouquet theorem [7,8]. Vesnitskiy, Metrikine and Belotserkovskiy employed a so-called periodicity condition [9–11]. More references on this subject can be found in the book of Frýba [12].

As compared to the above-mentioned studies, this paper treats an elastic system, which has not one but two spatial periods. The larger period is introduced by the fixations of the carrying cable, whereas the smaller period is associated with the droppers. Additionally, the two-level catenary combines two strongly coupled elastic systems (strings) with different wave speeds. To the authors' knowledge, the steady-state response of such a system to a moving load has not been studied in the past.

The 'periodicity condition method' is applied in this paper, although the other methods could be applied as well. This method is chosen since, in the opinion of the authors, it is elegant, can be interpreted physically, can be generalized to analyze three-dimensional periodically inhomogeneous systems [13,14] and even can be used to find the steady-state response of a layer of regularly positioned discrete particles [15].

This paper is structured in the following manner. In Section 2, a system of equations is presented that governs the transverse motion of the two-level catenary. This system is then transformed into the frequency domain, in which the steady-state solution is obtained in a closed form by employing the periodicity condition. This solution can be transformed into the time domain by using any conventional numerical-inversion technique. In Section 3, results are presented of the numerical analysis of the displacement of the contact cable and of the contact force between a hanger and this cable. In Sections 4 and 5, the simplified models for the catenary are introduced and the steady-state solutions for both these models are obtained in the frequency domain. In Section 6 a comparative study is carried out of the predictions of the original two-level model to those of the simplified models. Section 7 presents main conclusions, which can be drawn from this study.

2. Governing equations and the solution in the frequency domain

The model for a two-level catenary is composed of two parallel, infinitely long strings as depicted in Fig. 1. The upper string (the carrying cable) is fixed at periodically spaced fixation points $x = mD + d/2$, $m = 0, \pm 1, \pm 2, \ldots$, whereas the lower string (the contact wire) is suspended from the upper string by means of lumped mass-spring-dashpot elements (the suspension rods), which are placed periodically at $x = nd$, $n = 0, \pm 1, \pm 2, \ldots$ along the strings. The system in question is subject to a point load (the current collector),