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Solutions to the compact debris flight equations

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A R T I C L E I N F O

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A B S T R A C T

Analytic, approximate analytic, and numerical solutions to the compact debris flight equations are presented. Analysis shows that, after release from rest, the slope of the particle trajectory adjusts from an initial slope of $-\Omega$ to a final slope of $-\sqrt{\Omega}$ where Ω is the inverse of the Tachikawa number. For $\Omega < 1$ the trajectory steepens whereas for $\Omega > 1$ the path becomes less steep over the full flight distance. However, for all values of Ω the trajectory initially steepens before adjusting to its steady trajectory slope. The final steady straight line trajectory is shown to project back to a virtual release height that is calculated numerically and shown to be a function of Ω. Approximate analytical solutions for the flight distance required to achieve the final steady-state slope ($-\sqrt{\Omega}$) are presented and show that the transition height is a function of Ω for small values of Ω but is independent of Ω for larger values. The transition height is shown to be very large for a broad range of physically realistic conditions. Contour plots are presented that summarize the change in trajectory, horizontal flight distance, horizontal and vertical velocity, and kinetic energy as a function of vertical distance traveled and Ω.

1. Introduction

The flight of wind-borne compact debris during severe storms is of significant interest to the wind engineering community due to the severe damage that can be caused by such debris upon impact. A recent example of such damage was the destruction of a façade of the Hyatt Hotel in down town New Orleans due to roof gravel from an adjacent building being blown off during hurricane Katrina. It is therefore important to understand how far a piece of compact debris will travel for a given wind speed. The compact debris equations of motion have long been established (Tachikawa, 1983, 1988). However, there are no general analytic solutions to these equations so flight calculations must be done numerically which inhibits their general use.

The compact debris flight equations are mainly used in probabilistic models to assess risk that use statistical input and wind conditions. They are of limited use in predicting the flight dynamics of a given piece of debris as the size of the debris, the debris initial velocity, and the ambient wind field are all unknown. In general, the equations have been solved for a piece of compact debris released from rest at some specified height in a steady uniform wind field (Holmes, 2004; Baker, 2007). The goal with this approach is to understand the behavior of the system of equations under idealized conditions to provide a framework for interpreting the results of more sophisticated models. This approach has been extended to examine the role of ambient turbulence on the particle flight path (Holmes, 2004; Karimpour and Kaye, 2012a; Moghim and Caracoglia, 2014). Karimpour and Kaye (2012a) showed that ambient turbulence will slightly increase the flight distance and can be accounted for by using the Root Mean Squared horizontal wind speed in calculations. Karimpour and Kaye (2012a) also examined the role of input uncertainty on the debris flight path and showed that, for some set of randomly distributed particle sizes, using the mean particle size in flight calculations will underestimate the mean particle flight path.

Some special cases of the debris flight equations can be solved analytically. Holmes (2004) presented a solution for the case where vertical air resistance is ignored. Baker (2007) showed that, for long enough flight times, a piece of compact debris will reach a steady velocity in which it travels horizontally at the wind speed (U) and vertically at its terminal velocity (w_T). Therefore, provided the flight duration is long enough that the initial adjustment to steady-state can be ignored, the flight distance (X) for a particle released from rest at a height (H) above the ground, can be approximated by

$$X \approx HU/w_T. \tag{1}$$

However, no limitations on the use of this equation, or discussion of, in general, how large H must be for this to be a valid approximation, have been presented in the literature.

The solution of the compact debris equations has implications for a range of wind engineering applications. For example, theoretical estimates of flight distance and velocity could allow engineers to develop appropriate impact mitigation designs. Calculations of debris kinetic energy can be used to develop test standards for impact

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resistant cladding. Finally, the flight path of a piece of compact debris has implications for the scaling of wind tunnel blow-off tests up to full scale.

The remainder of the paper is structured as follows. The compact debris flight equations are presented in Section 2 in both dimensional and non-dimensional form along with the solutions for the near field and far field trajectory. The near field adjustment to the far field steady-state trajectory and the distance required to achieve that steady-state trajectory are discussed in Section 3. Full numerical solutions for a very broad range of Tachikawa number are presented graphically in Section 4. Conclusions are presented in Section 5.

2. Model development

The two-dimensional compact debris flight equations have been presented numerous times in the literature (Holmes, 2004; Baker, 2007). Compact debris is any object for which all length to width ratios are approximately one (as opposed to rod like debris which has one long and two short dimensions and plate like debris which has two long dimensions and one short dimension). Additionally, compact debris has a negligible lift coefficient and negligible rotational inertia. As such, the compact debris flight equations can be developed from the aerodynamic drag equation and the two dimensional equations of motion for a particle in a gravitational field. Consider a particle moving horizontally with velocity u and vertically with velocity $w<0$ (taking up as positive) in a steady uniform wind field of horizontal velocity U and zero vertical velocity (see Fig. 1a). The resulting drag force acts in the direction of the relative velocity while the weight force acts vertically downward (see Fig. 1b). Ignoring the buoyancy force acting on the particle then the resulting equations for the time variation of vertical and horizontal particle velocity are given by

$$\frac{d^2x}{dt^2}=\frac{du}{dt}=\frac{\rho C_D A}{2m}(U-u)\sqrt{(U-u)^2+w^2} \tag{2}$$

and

$$\frac{d^2z}{dt^2}=\frac{dw}{dt}=\frac{\rho C_D A}{2m}(-w)\sqrt{(U-u)^2+w^2}-g \tag{3}$$

where x and z are the horizontal and vertical coordinates, ρ is the density of air, m is the mass of the particle, A is the cross sectional area of the particle (assumed constant), C_D is a drag coefficient (assumed constant), and g is the gravitational acceleration constant.

These equations can be re-written in non-dimensional form as

$$\frac{d^2\chi}{d\tau^2}=\frac{d\mu}{d\tau}=(1-\mu)\sqrt{(1-\mu)^2+\omega^2} \tag{4}$$

and

$$\frac{d^2\zeta}{d\tau^2}=\frac{d\omega}{d\tau}=(-\omega)\sqrt{(1-\mu)^2+\omega^2}-\Omega \tag{5}$$

where the non-dimensional variables are given by

$$\mu=\frac{u}{U},\ \omega=\frac{w}{U},\ \chi=x\frac{C_D\rho A}{2m},\ \zeta=z\frac{C_D\rho A}{2m},\ \tau=t\frac{C_D\rho UA}{2m}\ \text{and}\ \Omega=\frac{2mg}{C_D\rho AU^2} \tag{6}$$

Here, Ω is proportional to the inverse of the Tachikawa number (Holmes et al., 2006). Note that this is a slightly different non-dimensional scheme than that used by Baker (2007) as the drag coefficient is included in the non-dimensionalization in order to keep the equations and resulting solutions tidier.

For a particle released from rest the initial conditions are zero horizontal and vertical particle velocity ($\mu=\omega=0$ at $\tau=0$). Therefore, the velocity diagram has only the horizontal wind speed U (see Fig. 2a) and the forces acting on the particle are the weight acting vertically down and the drag acting horizontally (see Fig. 2b). For small times after the initial release the equations of motion can be approximated by ignoring the particle velocity terms, giving

$$\frac{d^2\chi}{d\tau^2}=\frac{d\mu}{d\tau}=1\ \text{and}\ \frac{d^2\zeta}{d\tau^2}=\frac{d\omega}{d\tau}=-\Omega. \tag{7}$$

Therefore, the small time velocities are given by

$$\mu\approx\tau\ \text{and}\ \omega\approx-\Omega\tau, \tag{8}$$

leading to an initial trajectory slope of

$$S=\frac{\omega}{\mu}=-\Omega=-\frac{2mg}{C_D\rho AU^2} \tag{9}$$

equal to the initial force direction as shown in Fig. 2(b). This is similar to the result of Baker (2007), though again with the C_D contained in the non-dimensional parameter Ω.

In the limit of large time a steady trajectory is achieved in which the time derivatives in (4) and (5) are zero. This leads to the result that

$$\mu=1\ \text{and}\ \omega=-\sqrt{\Omega}. \tag{10}$$

That is, the debris travels horizontally at the wind speed (Fig. 2c). Therefore, the only drag force is in the vertical direction and is exactly balanced by the particles weight (Fig. 2d). As such,

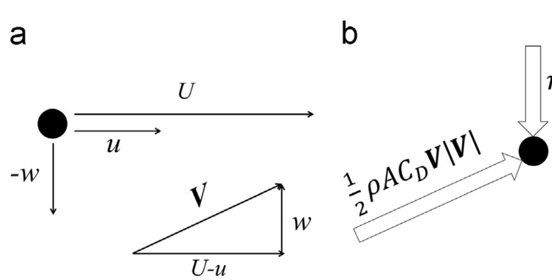

Fig. 1. (a) Velocity diagram for a particle showing the wind speed U, particle velocity (u,w) and velocity of the air relative to the particle V. (b) Force diagram showing the drag force acting in the direction of the relative velocity and the weight force acting down.

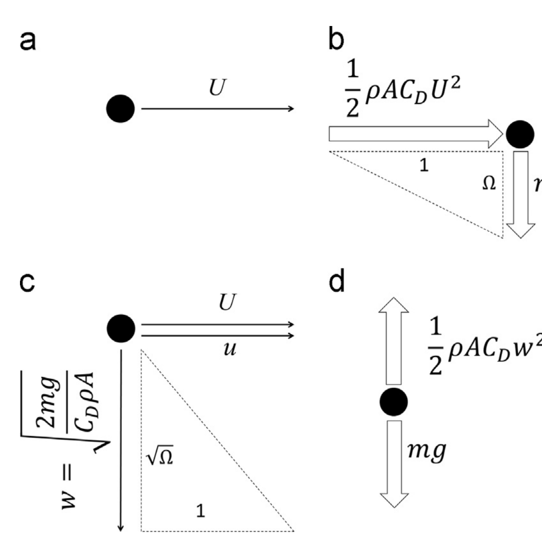

Fig. 2. Particle velocity diagrams (a,c) and force diagrams (b,d) for a piece of compact debris upon release (a,b) and at the large time limit steady state (c,d). The initial trajectory slope of $-\Omega$ is shown on the force diagram (b) and the final trajectory slope of $-\sqrt{\Omega}$ is shown on the velocity diagram (c).