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A modification to the flutter derivative model

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ABSTRACT

There is an ambiguity in the original equations for the identification of flutter derivatives. The analysis in this paper shows that, for the two degrees of freedom (2DOF) model, these derivatives in these equations are, in fact, three-dimensional functions instead of one dimensional functions of reduced frequency. Alternative governing equations are proposed in this paper to solve this problem. These new equations “decouple” the aeroelastic coupling effect by approximated modeling strategy. The number of independent aeroelastic parameters is reduced from eight to five. Nonlinear nature of the aeroelastic coupling effect is unveiled. The nonlinear identification method is presented to retrieve the new set of aeroelastic parameters. Wind tunnel experiments were conducted to show the effectiveness of the newly proposed equations.

1. Introduction

Flutter derivatives (Scanlan and Tomko, 1971) in bridge aeroelasticity correspond to aerodynamic coefficients (Theodorsen, 1934) in airfoil aeroelasticity.

In the case of thin airfoil, the shape of the structural cross section is carefully designed to avoid the separation of flow from the structural surface. Under the condition of small attacking angle, the potential flow theory can be utilized and linearity of the fluid behavior is assumed.

The bridge decks, on the other hand, has to be bluff, and cannot be analyzed effectively by potential flow theory. It has not to date been possible to develop analytical expressions for the flutter derivatives from basic fluid dynamics. Wind tunnel experiments are usually relied upon to identify these parameters. Sectional model tests are routine practices in this area (Chen et al., 2010; Mishra, 2006; Chowdhury and Sarkar, 2004; Zhang and Brownjohnb, 2004; Li et al., 2003; Gu et al., 2000; Iwamoto and Fujino, 1995; Jakobsen and Hansen, 1995; Sarkar et al., 1994).

2. The flutter derivative model and its ambiguity

2.1. The flutter derivative model

The sectional model used in wind tunnel experiments is usually assumed to have vertical, rotational and lateral degrees of freedom (DOFs). Two-DOF (vertical-rotational) model is commonly used. The sectional model is assumed to rotate about the structural elastic center (SEC). Governing equations of the vibration is thus established as (Simiu and Scanlan, 1996):

$$m\ddot{h}+S\ddot{\alpha}+c_h\dot{h}+k_h h=L_{ae} \tag{1}$$

$$S\ddot{h}+I\ddot{\alpha}+c_\alpha\ddot{\alpha}+k_\alpha\alpha=M_{ae} \tag{2}$$

where h and α are the motions in vertical and rotational DOF, respectively; m and I are the mass and moment of mass, respectively; S is the static unbalance, i.e. the product of mass and the distance separating the mass center and structural elastic center. S equals to zero when the model is symmetric. The aeroelastic forces in (1) and (2) are

$$L_{ae}=\frac{1}{2}\rho U^2(2\mathrm{B})\left[KH_1^*(\mathrm{K})\frac{\dot{h}}{U}+KH_2^*(K)\frac{(2B)\dot{\alpha}}{U}+K^2H_3^*(K)\alpha+K^2H_4^*(K)\frac{h}{(2B)}\right]; \tag{3}$$

$$M_{ae}=\frac{1}{2}\rho U^2(2B)^2\left[KA_1^*(K)\frac{\dot{h}}{U}+KA_2^*(K)\frac{(2B)\dot{\alpha}}{U}+K^2A_3^*(K)\alpha+K^2A_4^*(K)\frac{h}{(2B)}\right], \tag{4}$$

where H_i^* and A_i^* , $i=1,2,3,4$ are flutter derivatives; $K=((2B)\omega/U)$ is the reduced frequency; B is the half deck width; U is the wind speed and ω is the vibration circular frequency.

The flutter derivative model is the corner stone of bridge aeroelasticity. However, there is some ambiguity in the expressions.

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Nomenclature

$A_{hp}(t)$ The Hilbert amplitude of $h_p(t)$
A_i^*, $i=1,2,3,4$ flutter derivatives
B half deck width
B_s half separation between springs
C coefficient
c_{h_ae}, c_{α_ae} aeroelastic damping of vertical and rotational DOF, respectively
c_{h_s}, c_{α_s} structural damping of vertical and rotational DOF, respectively
D coefficient
$e_r(t)$ aeroelastic eccentricity
$h_p(t)$, $h(t)$ vertical displacement measured at aeroelastic center and structural elastic center, respectively
H_i^*, $i=1,2,3,4$ flutter derivatives
I_α, I_o moment of inertia about aeroelastic center and structural elastic center, respectively
K the reduced frequency
k_{h_s}, k_{α_s} spring stiffness of vertical rotational DOF, respectively
k_{h_ae}, k_{α_ae} aeroelastic stiffness of vertical and rotational DOF, respectively
L_{ae}, M_{ae} aeroelastic lifting force and moment, respectively
m mass
T kinetic energy
U wind speed
V potential energy
$\alpha(t)$ rotational displacement
ω_{hp_d}, ω_{α_d} damped circular frequency of the signal

2.2. The ambiguity of the flutter derivative model

On the one hand, Eqs. (1) and (2) indicate h and α should both be double frequency signals, otherwise, when the static unbalance is zero ($S=0$), i.e. when the model is symmetric, (1) and (2) would have single frequency on the left hand side and two frequencies on the right, resulting in unbalanced equations. On the other hand, for Eqs. (3) and (4) to be physically meaningful, h and α should both be single frequency signals. This is because flutter derivatives are assumed to be one dimensional functions of reduced frequency; they may fail to be effective if the signals associated with them contain multi-modal components.

Suppose the translational motion, for instance, contains two frequency components: the translational motion with the frequency of vertical DOF, h_h and the translational motion with the frequency of rotational DOF, h_α. It can be decomposed in the following manner:

$$h=h_h+h_\alpha. \tag{5}$$

Following the decomposition in (5), the aeroelastic forces due to h may also be decomposed as

$$L_{ae\_h}=\frac{1}{2}\rho U^2(2B)\left[K_{\omega h}H_1^*(K_{\omega h})\frac{\dot{h}_h}{U}+K_{\omega\alpha}H_1^*(K_{\omega\alpha})\frac{\dot{h}_\alpha}{U}+K_{\omega h}^2H_4^*(K_{\omega h})\frac{h_h}{(2B)}+K_{\omega\alpha}^2H_4^*(K_{\omega\alpha})\frac{h_\alpha}{(2B)}\right], \tag{6}$$

$$M_{ae\_h}=\frac{1}{2}\rho U^2(2B)^2\left[K_{\omega h}A_1^*(K_{\omega h})\frac{\dot{h}}{U}+K_{\omega\alpha}A_1^*(K_{\omega\alpha})\frac{\dot{h}_\alpha}{U}+K_{\omega h}^2A_4^*(K_{\omega h})\frac{h_h}{(2B)}+K_{\omega\alpha}^2A_4^*(K_{\omega\alpha})\frac{h_\alpha}{(2B)}\right]. \tag{7}$$

It is important to note that $H_1^*(K_{\omega h})$, $H_4^*(K_{\omega h})$, $A_1^*(K_{\omega h})$ and $A_4^*(K_{\omega h})$ are valued at reduced frequency $K_{\omega h}$, while $H_1^*(K_{\omega\alpha})$, $H_4^*(K_{\omega\alpha})$, $A_1^*(K_{\omega\alpha})$ and $A_4^*(K_{\omega\alpha})$ are valued at reduced frequency $K_{\omega\alpha}$, where $K_{\omega h}\neq K_{\omega\alpha}$.

What is indicated by (6) is that, H_1^* and H_4^* in original Eq. (3) reflect the effect of the weighted summation of two motions of different frequencies and amplitudes. In this case, the identified flutter derivatives in the traditional manner will be a function of two reduced frequencies and the ratio of amplitudes between h_h and h_α. They are indeed three-dimensional functions! This is clearly not the original intention of the flutter derivative model. The same statement is true for other flutter derivatives as well.

However, it is not feasible, to utilize equations like (6) and (7) for the identification of flutter derivatives, because this will results in more parameters to be identified than the traditional case, creating a new challenge to the identification method. The modification of the flutter derivative model will have to take another direction.

In the following part, we derive alternative governing equations for the bridge model on the condition that reasonable approximations are tolerated. It can be seen the ambiguity is partly clarified by the proposed equations.

3. Alternative governing equations for the free vibration of bridge deck in wind

3.1. An approximated modeling strategy

In the derivation of aerodynamic coefficients (Theodorsen, 1934), wing's motion is prescribed as $h=h_0e^{j\omega t}$; $\alpha=\alpha_0e^{j\omega t}$, i.e. a coupled sinusoidal motion with h and α components. The frequencies of vertical and rotational vibrations are the same.

However, under the operational condition of sectional model test for bridges, it is impractical to create the motions in this way. The initial displacement assigned to the model will always create double frequency responses (for 2DOF model) due to the aeroelastic coupling effect.

However, the aeroelastic coupling affects the vertical and rotational motions in different ways. On one hand, both the rotational displacement and rotational velocity have effects on the aeroelastic lifting force. On the other hand, only the vertical velocity affects the aerodynamic moment while the vertical position (displacement) has no role to play in generating the aeroelastic moment. Because there is a phase lag between the vertical velocity and the aeroelastic moment generated by the velocity, it appears that the moment can be expressed, under the condition of sinusoidal motion, as a function of velocity and displacement. This does not mean the aeroelastic moment is generated by the vertical displacement. Therefore, while the aeroelastic coupling produces both stiffness and damping effect in the response of vertical DOF, it may only create aeroelastic damping effect in the response of rotational DOF.

In view of this, it can be said that the vertical response due to aeroelastic coupling is both stiffness-driven and damping-driven while the rotational response due to aeroelastic coupling is only damping-driven. Besides, the damping-driven responses initiate from zero displacement and velocity condition, they take time to build up before the triggered free vibrations die out. Therefore, it is reasonable to assume the coupled component in rotational motion is usually not very strong compared with the triggered rotational