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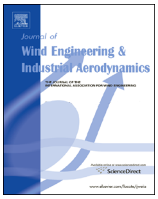

Vorticity generation and wake transition for a translating circular cylinder: Wall proximity and rotation effects

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ABSTRACT

The wake transitions of generic bluff bodies, such as a circular cylinder, near a wall are important because they provide understanding of different transition paths towards turbulence, and give some insight into the effect of surface modifications on the flow past larger downstream structures. In this article, the fundamentals of vorticity generation and transport for the two-dimensional flow of incompressible Newtonian fluids are initially reviewed. Vorticity is generated only at boundaries by tangential pressure gradients or relative acceleration. After generation, it can cross-annihilate with opposite-signed vorticity, and can be stored at a free surface, thus conserving the total vorticity, or circulation. Vorticity generation, diffusion and storage are demonstrated for a cylinder translating and rotating near a wall. The wake characteristics and the wake transitions are shown to change dramatically under the influence of cylinder rotation and wall proximity. At gaps between the cylinder and the wall of less than approximately 0.25 cylinder diameter, the wake becomes three dimensional prior to becoming unsteady, while for larger gaps the initial transition is to an unsteady two-dimensional wake. At a gap of 0.3 cylinder diameter, we observe a sharp increase in the critical Reynolds number at which three-dimensionality sets in. As the gap is further increased, the critical Reynolds number initially decreases before increasing to that for an isolated cylinder. The effect of cylinder rotation on these transitions is also quantified, with forward (prograde) rotation enhancing three-dimensional instability and reverse (retrograde) rotation stabilising the wake. High retrograde rotation leads to suppression of three-dimensional flow until beyond the highest Reynolds number investigated ($Re=750$).

1. Introduction

Fundamental to understanding flows around, and forces on, stalled airfoils and bluff bodies in aeronautics and wind engineering are the wake flow structures and the transitions between different wake modes, which substantially control wake development. The circular cylinder is a generic bluff body that has been used extensively to gain insight into the effect of these transitions on the wake. The classic theoretical model is an infinite cylinder, in practice experimentally modelled by high aspect ratio cylinders in wind or water tunnels (and, of course, cylindrical geometry is clearly the basis of the design of tall buildings for very high Reynolds number flows). The current paper focusses on much lower Reynolds number flows, with direct relevance to understanding wall-particle interactions that commonly occur in industrial processes such as sedimentation and mixing tanks, heat exchangers, etc). The Reynolds number for these latter applications is $Re=UD/\nu=O(10^2\text{–}10^3)$, where U is the velocity, D is the characteristic length and ν is the kinematic viscosity of the fluid. Experimental results at higher Reynolds numbers often show that the initial three-dimensional instability modes for these simplified geometries are still present and strongly influence the fully turbulent flow. For instance, *mode B*, which appears in a cylinder wake at $Re \approx 230$ persists and is observable at much higher Reynolds number when the flow is fully turbulent (e.g., Wu et al., 1996), whilst the Strouhal number of the wake remains approximately constant over the range $200 \le Re \le 10^5$ (e.g., Batchelor, 1967). Thus, as well as their direct relevance to specific industrial applications, the results from such studies may eventually assist with interpreting higher Reynolds number flows in vehicular aerodynamics, flow past low rise buildings, surface

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roughness effects of mounted structures over roofs of buildings, etc.

Vorticity is one of the most important physical quantities in fluid mechanics used to characterise a flow. Boundary layers, wakes and turbulence in bluff body flows owe their presence to, and are essentially defined by, vorticity and vortices, whose motions are associated with fluctuating forces in a flow, including those leading to VIV (Vortex induced vibration). To understand wake structures and transitions, and how they can be controlled, it is important to determine and understand the mechanism of vorticity generation and its subsequent evolution. In this study, we focus on the canonical geometry of a circular cylinder to gain an understanding of the variety of wake structures and transitions that arise as the influence of both wall proximity and cylinder rotation are increased.

The flow past a circular cylinder in freestream has been widely investigated through both experimental (Williamson, 1988, 1996; Roshko, 1954) and numerical techniques (Karniadakis and Trintafyllou, 1992; Thompson et al., 1996; Barkley and Henderson, 1996). At low Reynolds numbers, steady flow is observed with the formation of two recirculation regions in the near wake until $Re \simeq 47$. Beyond this Reynolds number, unsteady flow is observed. Vortices are alternately shed into the wake from the cylinder leading to the classical Bénard–von Kármán wake. Around $Re = 190$, the flow becomes three-dimensional with a spanwise wavelength of approximately four cylinder diameters; this instability is commonly referred to as the *mode A* instability (Barkley and Henderson, 1996; Williamson, 1988, 1996; Thompson et al., 1996). At slightly higher Reynolds number, $Re \simeq 230$–240, a short wavelength instability develops with a spanwise wavelength of approximately one diameter on the already three-dimensional wake, and this has been termed *mode B*. The equivalent two transitions to three-dimensional flow have been observed in other bluff bodies wakes such as for square and elliptical cylinders, which undergo these transitions at slightly lower Reynolds numbers.

For a rotating circular cylinder in freestream, the onset of vortex shedding depends on the combination of the Reynolds number and the rotation rate, α, where $\alpha = \omega D/2U$, the ratio of the surface tangential velocity to the freestream flow speed. The critical Reynolds number at which periodic flow is observed is delayed to higher Reynolds number as the rotation rate is increased (Kang et al., 1999). For $\alpha \gtrsim 2$, vortex shedding was suppressed even at high Reynolds numbers (Akoury et al., 2008; Pralits et al., 2010) and the flow remains steady. However, at still higher rotation rates, a secondary shedding regime was observed for $\alpha \gtrsim 4.35$ (Mittal and Kumar, 2003; Stojković et al., 2003; Akoury et al., 2008; Kumar et al., 2011). In this shedding regime, single-sided vortex shedding occurs by release of positive vorticity into the wake at a low shedding frequency. For $\alpha \gtrsim 5.5$, the flow remains steady. For a rotating cylinder, the onset of three-dimensionality has been well documented in a recent investigation by Rao et al. (2013) for $\alpha \le 2.5$. While mode A and mode B instabilities are delayed to higher Reynolds numbers at low rotation rates, a subharmonic mode becomes unstable for $\alpha \simeq 1.5$. Two other three-dimensional modes, whose spatio-temporal characteristics are similar to the mode A instability, occur on the unsteady base flow, and two other modes are observed on the steady base flow for $\alpha \gtrsim 2$.

Although the flow structures and the forces on a body in isolation have been well documented, fewer investigations have been performed for bodies near a plane boundary. One of the earliest investigations for bodies moving near a wall was made by Taneda (1965), who observed the flow structures for a circular cylinder translating along a wall at $Re = 170$. For the cylinder very close to a wall ($G/D = 0.1$), a single-sided vortex street was observed. Here, G/D is the non-dimensionalised gap height, where the distance between the cylinder and the wall is G and the cylinder diameter is D. The vortices were unstable and diffused as they convected downstream.

Experimental investigations were performed by Bearman and Zdravkovich (1978) for a cylinder near a fixed wall at $Re = 4.5 \times 10^4$ for $0 \le G/D \le 3.5$. The cylinder was located approximately $36D$ from the start of a turbulent boundary layer, which developed along the wall. They observed the suppression of regular vortex shedding for $G/D < 0.3$, with the Strouhal number remaining constant until this gap height was approached. Bailey et al. (2002) performed experimental investigations for a square cylinder near a stationary wall at $Re = 18{,}900$ and observed that the flow becomes increasingly two-dimensional in the range $0.53 \le G/D \le 0.7$.

Using a finite-difference method, Lei et al. (2000) performed numerical simulations for a circular cylinder for gap heights between $0.1 \le G/D \le 3$ and Reynolds numbers between $80 \le Re \le 1000$. The frame of reference was such that the flow moved past the fixed lower wall and the cylinder, leading to the development of a boundary layer ($16D$ upstream of the cylinder). They observed that the gap height at which vortex shedding was suppressed decreased as the Reynolds number was increased up to $Re = 600$. Beyond this value, the critical gap height remained constant. At higher Reynolds numbers, Price et al. (2002) observed periodicity in the upper shear layer for all gap heights $G/D > 0.125$. While the pairing of shear layers from either side of the cylinder was observed for $0.25 \le G/D \le 0.375$, vortex shedding was observed at higher gap heights of $G/D > 0.5$.

Mahir (2009) investigated the onset of three-dimensional flow for a square cylinder near a fixed wall for $Re \le 250$ as the gap height was increased from $0.1 \le G/D \le 4$. At $Re = 185$, mode A type vortex structures with spanwise spacing of $\simeq 3D$ were observed for gap heights greater than $G/D = 1.2$, while at $G/D = 0.8$, mode B type vortex structures of $1D$ spanwise wavelength were observed. Below $G/D = 0.5$, neither mode A nor B type vortex structures were observed. At $Re = 250$, mode B type vortex structures were observed at larger gap heights, while at lower gap heights, the vortex structure was distorted in the vicinity of the cylinder.

Cheng and Luo (2007) investigated the flow structures and forces on a rotating circular cylinder near a wall at $Re = 200$. As the cylinder was moved away from the wall, they observed the suppression of vortex shedding ($G/D \simeq 0.5$), followed by a region of aperiodic vortex shedding and finally a region of alternate Bénard–von Kármán shedding ($G/D \simeq 1.5$). They further quantified the lift and drag coefficients experienced by the cylinder.

Experimental investigations were performed by Nishino et al. (2007) for a circular cylinder near a moving wall for higher Reynolds numbers ($O(10^5)$). For a cylinder with endplates, they reported that the flow essentially remained two-dimensional, with Bénard–von Kármán type vortices being shed for gap heights $G/D > 0.5$, and an intermediate shedding regime was observed in the range $0.35 \le G/D \le 0.5$. Complete cessation of shedding occurred for $G/D < 0.35$. However, for a cylinder without endplates, they reported that Bénard–von Kármán type wake vortices were not being generated, and a near constant drag coefficient was recorded.

Huang and Sung (2007) performed two-dimensional simulations for a circular cylinder moving near a wall for gap heights in the range $G/D > 0.1$ for $Re \le 600$. The gap heights at which alternate vortex shedding disappeared decreased from $0.28D$ to $0.25D$ as the Reynolds number was increased from 300 to 600. The non-dimensionalised shedding frequency ($St = fD/U$, with f the shedding frequency) at different Reynolds numbers increased as the cylinder was brought close to the wall ($\simeq 0.5D$), followed by a rapid decrease as the gap height was decreased. They further quantified the lift and drag coefficients, with the lift coefficient showing a linear increase as the cylinder was brought closer to the wall. They however acknowledged their restriction of two-