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Stochastic differential calculus for wind-exposed structures with autoregressive continuous (ARC) filters

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ABSTRACT

In this paper, an alternative method to represent Gaussian stationary processes describing wind velocity fluctuations is introduced. The technique may be considered the extension to a time continuous description of the well-known discrete-time autoregressive model to generate Gaussian processes. Digital simulation of Gaussian random processes with assigned auto-correlation function is provided by means of a stochastic differential equation with time delayed terms forced by Gaussian white noise. Solution of the differential equation is a specific sample of the target Gaussian wind process, and in this paper it describes a digitally obtained record of the wind turbolence. The representation of wind fluctuations with the proposed model is suitable for the use of stochastic differential calculus in wind-engineering applications. Some numerical applications dealing with structural models in presence of the wind fluctuations have been reported to challenge the robustness of the proposed method in the representation of stationary random process of wind-turbolence and its accuracy for engineering analysis.

1. Introduction

Wind engineering analysis involving the evaluation of the structural response in presence of wind pressure has been extensively investigated in the last years assuming stochastic models of speed fluctuations. In this context, the widest used methods to analyze wind-exposed structures are the Monte-Carlo techniques or the frequency-domain analysis. On one hand, Monte-Carlo methods may

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be used to yield a reliable digital simulation of the wind speed fluctuation records with prescribed, experimentally estimated power spectrum. On the other hand, robust Monte-Carlo analysis of structural systems under external or parametric-type excitations requires several thousands, sometimes hundred of thousands, of digital sample generations yielding prohibitive computational efforts.

As an alternative closed-form, frequency-domain analyses have been proposed with preliminar linearization of the dynamic equilibrium equation. In this context the obtained statistics are often affected by significative errors.

Engineering analyses of wind-exposed structures in the context of Monte-Carlo methods require efficient techniques to generate digital wind turbolence usually modeled as Gaussian and stationary random process (Maeda and Makino, 1992; Smith and Mehta, 1993; Huang and Chalabi, 1995). In this setting recursive algorithms to generate digital samples such autoregressive (AR), or moving average autoregressive methods (ARMA) have been proposed (Gersch and Liu, 1976; Samaras et al., 1985; Spanos and Mignolet, 1987a, b; Mignolet and Spanos, 1990). The recursive simulation of random processes within AR or ARMA models shows some difficulties in presence of peculiarity of the second-order statistics, namely the autocorrelation function (CF) or the power spectral density (PSD) function of the target process (Mignolet and Spanos, 1991). Moreover, the recursive representation of wind-speed fluctuations yields non-white random process and it does not allow the analysis with the aid of stochastic differential (Itô) calculus (Itô, 1951). This latter tool has proved to be an efficient method to evaluate the statistics of linear and non-linear dynamical systems under external and parametric white noises including Poissonian and Lèvy delta-correlated excitations (Di Paola and Falsone, 1983; Pirrotta, 2005; Di Paola et al., 2007). Stochastic differential calculus has been extended to analysis in presence of non-white excitations introducing the so-called "filtering theory" (Anderson and Moore, 1979; Jazwinski, 1970). In this framework the non-white excitation with prescribed statistics is obtained as the output of a set of linear differential equations enforced by a Gaussian white noise. In the context of wind engineering some contributions to the stochastic dynamics by the use of filtering theory has been often found in scientific literature (Bartoli and Spinelli, 1990; Benfratello and Muscolino, 1999; Naess, 2001). However, the filtering theory has proved to be non-satisfactorily for wind-engineering applications since the coefficients of system of differential equations are hardly evaluated with the requirement to yield the prescribed statistics of the excitation (Gullo et al., 1998).

In this paper a quite different approach, bridging the positive aspects of filtering theory and of recursive simulation of Gaussian and stationary random processes, is proposed. The methodology may be considered as the continuous counterpart of the AR models, dubbed in the following autoregressive continuous (ARC) filters. The stationary and Gaussian random processes obtained via ARC filters of order m are described by the output of a linear differential equations with m delayed terms enforced by external white-noise excitation. The coefficient of the differential equation may be evaluated via the well-known Yule–Walker scheme involving a linear system of m algebraic equations in m unknown coefficients.

The digital generation of Gaussian stochastic process obtained with the ARC models provides second-order statistics in good agreement to the target used in wind engineering applications. This aspect has been investigated in this paper analyzing some widely used power spectra describing wind fluctuations. With this machinery in mind the stochastic differential calculus for wind-exposed structures may be used to evaluate the statistics of the structural response (linear or non-linear) avoiding Monte-Carlo analysis. The same technique may be used to model the multivariate wind pressure distribution by using proper orthogonal decomposition of the PSD matrix (Di Paola and Zingales, 2000). Some numerical applications have been reported to challenge the robustness and the accuracy of the ARC models in wind-engineering structural analysis.

2. The autoregressive continuous (ARC) filter

Let us consider a stationary and Gaussian random process $V(t)$, with zero mean and prescribed autocorrelation function $R_V(\tau)$, with τ the time lag, or as an alternative, with selected PSD function