HOSTED BY
ELSEVIER
Available online at www.sciencedirect.com
ScienceDirect
journal homepage: www.elsevier.com/locate/jtte

Original Research Paper

Analysis of transient viscoelastic response of asphalt concrete using frequency domain approach

Yanqing Zhao*, Sajjad Yousefi Oderji, Peisong Chen

School of Transportation Engineering, Dalian University of Technology, Dalian 116024, China

ARTICLE INFO

Article history:
Available online 17 October 2015

Keywords:
Asphalt concrete
Linear viscoelastic
Transient response
Fourier transformation
Aliasing

ABSTRACT

The analysis of transient linear viscoelastic response of asphalt concrete (AC) is important for engineering applications. The traditional transient response of AC is analyzed in the time domain by performing complicated convolution integral. The frequency domain approach allows one to determine the transient responses by performing simple multiplication instead of the complicated convolution integral, and it does not require the time derivative of the input excitation, and thus, the approach could greatly reduce the analysis complexity. This study investigated the frequency domain approach in calculating the transient response by utilizing the discrete Fourier transform technique. The accuracy and effectiveness of the frequency domain approach were verified by comparing the analytical and calculated responses for the standard 3-parameter Maxwell model and by comparing the time and frequency domain solutions for AC. The effect of aliasing of the frequency domain approach can effectively reduce by selecting a small sampling interval for the time domain excitation function. A sampling interval is acceptable as long as the amplitude of the Fourier transformed excitation is close to 0 more than half of the sampling rate. The results show that the frequency domain approach provides a simple and accurate way to perform linear viscoelastic analysis of AC.

1. Introduction

Frequency domain analysis plays an important role in many branches of engineering. By taking the Fourier transform, the differential or integral equations used to describe a physical system in the time domain are converted to algebraic equations in the frequency domain, which greatly reduces the complexity of the problem-solving process. The Fourier transform is essentially a universal problem-solving technique (Brigham, 1988), and it allows one to examine a particular relationship from an entirely different viewpoint. Furthermore, the computational efficiency can be significantly improved by taking the advantage of the fast Fourier transform (FFT) algorithm.

In pavement engineering, an important application of the frequency domain approach is to determine the complex modulus of asphalt concrete (AC) (AASHTO, 2007). In the complex modulus testing, the specimen is subjected to a

* *Corresponding author*. Tel.: +86 15804269202.
E-mail addresses: yanqing_zhao@dlut.edu.cn (Y. Zhao), su_a_2006@yahoo.com (S. Y. Oderji), sdcwcps@163.com (P. Chen).
Peer review under responsibility of Periodical Offices of Chang'an University.
http://dx.doi.org/10.1016/j.jtte.2015.10.003

cyclic haversine stress or strain excitation, and the complex modulus is determined from the steady-state response (Chehab et al., 2003; Gibson et al., 2003; Pellinen et al., 2007; Underwood et al., 2006). Theoretically, the steady-state response can only be achieved after infinite load cycles for linear viscoelastic (LVE) solid materials. However, the in-situ response caused by a moving load is far from reaching the steady-state. Instead, the response decays gradually and approaches 0 as the load moves away. Therefore, the transient response analysis is more pertinent to engineering applications (Chen et al., 2009; Elseifi et al., 2006; Wang et al., 2006; Zhao et al., 2014).

The transient LVE response of AC is conventionally analyzed in the time domain through numerical evaluation of the convolution integral. Besides the computation complexity in solving the convolution integral, analysis in the time domain requires the derivative of input excitation with respect to time, which usually causes computational difficulties when the excitation contains discontinues. These shortcomings are avoided in frequency domain approach. Researchers have used Fourier transform or Laplace transform to conduct inter-conversions of LVE functions of AC. In these studies, theoretical solutions can be derived, since the time domain functions are defined as the responses due to constant stress or strain input (Kim et al., 2008; Luo and Lytton, 2010). This study aims to investigate the frequency domain approach in analyzing transient responses of AC under arbitrary input history where the theoretical solutions are not available. Also, methods for improving the accuracy of analysis results are proposed.

2. Theoretical background and discrete Fourier transform

According to the Boltzmann superposition principle, the constitutive relationship of LVE materials can be expressed in the form of convolution integral as follows

$$\sigma(t) = \int_0^t E(t-\tau)\frac{\partial \varepsilon}{\partial \tau}d\tau \tag{1}$$

$$\varepsilon(t) = \int_0^t D(t-\tau)\frac{\partial \sigma}{\partial \tau}d\tau \tag{2}$$

where ε and σ are strain and stress, respectively, τ is a time-like integration variable, t is time, $E(t)$ and $D(t)$ are relaxation modulus and creep compliance, respectively. By applying the Fourier transform to both sides of the above equations and after some mathematical manipulations, the following relations are obtained (Tschoegl, 1989).

$$\overline{\sigma}(\omega) = \overline{\varepsilon}(\omega)E^*(\omega) \tag{3}$$

$$\overline{\varepsilon}(\omega) = \overline{\sigma}(\omega)D^*(\omega) \tag{4}$$

where ω is the angular frequency, a bar over a symbol means that the quantity has been Fourier transformed, $E^*(\omega)$ and $D^*(\omega)$ are complex modulus and complex compliance, respectively, and their relationship is as follow

$$E^*(\omega) = \frac{1}{D^*(\omega)} \tag{5}$$

where $E^*(\omega)$ and $D^*(\omega)$ are also called frequency response functions (FRF). The Fourier transform of a time domain function, $f(t)$, is defined as follow

$$\overline{f}(\omega) = \int_{-\infty}^{+\infty} f(t)e^{-j\omega t}dt \tag{6}$$

where $j = \sqrt{-1}$, $\overline{f}(\omega)$ is a complex function and it has real and imaginary parts.

The Fourier transformed stress–strain relationship shown in Eqs. (3) and (4) allows one to determine the viscoelastic response by performing simple multiplication in the frequency domain instead of complicated convolution in the time domain. The evaluation of Eqs. (3) and (4) involves 3 steps: (1) transform the time domain excitation to the frequency domain; (2) multiply the transformed excitation by the corresponding FRF; (3) take the inverse Fourier transform of the multiplication results to obtain the time domain response solution. The inverse Fourier transform is defined as

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \overline{f}(\omega)e^{j\omega t}d\omega \tag{7}$$

For real-world engineering applications, the right sides of Eqs. (3) and (4) are usually complicated (and complex) function, and the analytical solution of the inverse transform is not easily obtained. In such cases, the forward and inverse transforms need to be evaluated using discrete Fourier transform (DFT).

To use DFT, a time domain function, $f(t)$, is sampled to obtain a discrete-time sequence denoted by $x(n) = (x_0, x_1, \cdots, x_{N-1})$, in which the variable n is integer-valued and represents discrete instances in time. It is noted that the index of $x(n)$ starts at 0 indicating the first sample time is 0. The DFT of the N-point sequence, $x(n)$, is given by Stearns and Hush (2003).

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi nk/N} \quad k = 0, 1, \cdots, N-1 \tag{8}$$

where $X(k)$ is a complex-valued sequence $(X_0, X_1, \cdots, X_{N-1})$, equally separated by $\Delta\omega$ on the frequency axis, $\Delta\omega$ is the frequency domain sample interval and $\Delta\omega = 2\pi/(\Delta t N)$. The frequency of X_i is mean $i\Delta\omega$. The inverse discrete Fourier transform (IDFT) of $X(k)$ is given by Stearns and Hush (2003).

$$X(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j2\pi nk/N} \tag{9}$$

Eqs. (8) and (9) can be efficiently evaluated through the fast Fourier transform (FFT) algorithm (Cooley and Turkey, 1965).

3. Analysis of standard 3-parameter Maxwell model

The standard 3-parameter Maxwell model was first analyzed to evaluate the effectiveness of the frequency domain