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Minimizing errors in interpolated discrete stochastic wind fields

Manuel Fluck*, Curran Crawford

Sustainable Systems Design Laboratory, Department of Mechanical Engineering, University of Victoria, 3800 Finnerty Road, Victoria, British Columbia, Canada

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ABSTRACT

For many unsteady processes (e.g. turbulent wind, electricity demand, traffic, financial markets, space physics, etc.) data is only available at discrete points, be it due to data storage or data gathering limitations. However, derived forms of that data are often used in further studies where the discretization may be different from the discretization of the original data. This paper addresses the question of how to obtain values between discrete data points, for example, when sampling turbulent wind. Linear interpolation is often the standard answer. Yet, it is shown that this is a poor choice for unsteady processes where the sample step size is significantly larger than the fluctuation scale. An alternative employing probability density functions of data increments is suggested. While this new method does not require much more effort than linear interpolation, it yields significantly more accurate results. Unsteady wind is used to exemplify this: turbulent wind speeds on a (rotating) wind turbine blade are synthesized from a coarse data grid via the introduced method of velocity increments. Thus the superiority of the presented approach over linear interpolation is demonstrated – with important implications for blade load and power output computations.

1. Introduction

In engineering application we often have to deal with unsteady, highly fluctuating processes. e.g. turbulent atmospheric wind, urban electricity demand, local traffic volume, financial markets, space physics, etc. Due to limitations in data handling and/or storage capacity often a full time series of the process under investigation is not available. Instead, only incomplete data sets at discrete points are at hand. However, these data sets are regularly used as input for further analysis and often data values between two sample points are needed. To solve this task, interpolation based on deterministic, continuous, and possibly multivariate algebraic or sometimes trigonometric polynomials, with the number of variables depending on the considered problem (one or several interpolation dimensions), is the common solution (Phillips, 2003; Steffensen, 2006; Mastroianni and Milovanovic, 2008). Linear interpolation is the most basic (and very widely used) example of this kind of interpolation in a one-dimensional space.

However, if the interpolation time and/or length scales are significantly larger than the signal's fluctuation scale, these interpolation schemes become erroneous. In fact, in these cases conventional interpolation with continuous functions acts as a low-pass filter. Thus it results in a reduced variance σ^2 of the interpolated signal, i.e. a reduced likelihood of extreme events, and consequently a distorted spectrum. Fig. 1 illustrates this. A set of $N=100$ data points, labelled 'original process', is considered as a generic example of some unsteady, highly fluctuating process with a short fluctuation scale. The N points were generated independently and standard normal distributed (variance $\sigma^2=1$). If this process is reconstructed via linear interpolation from a set of ten equidistant sample points, much further apart than the fluctuation scale, the resulting process (labeled 'interpolated') is obviously considerably smoother, and the signal variance drops to $\sigma^2=0.67$.[1] Clearly this results in an error when the interpolated data set is used in further analysis.

Although a simple Gaussian process was used here for illustration, linear interpolation obviously has the same effect on various kinds of weakly correlated processes with short fluctuation scales. Switching from linear to higher order interpolation methods might mitigate these effects and potentially even conserve the statistical moments (see e.g. Mastroianni and Milovanovic's discussion on moment-preserving approximation, Mastroianni and

* Corresponding author.
E-mail address: mfluck@uvic.ca (M. Fluck).

[1] For clarity only a short process is shown in Fig. 1. However, to achieve statistically stable results $N \gg 100$ data points were considered.

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0167-6105/

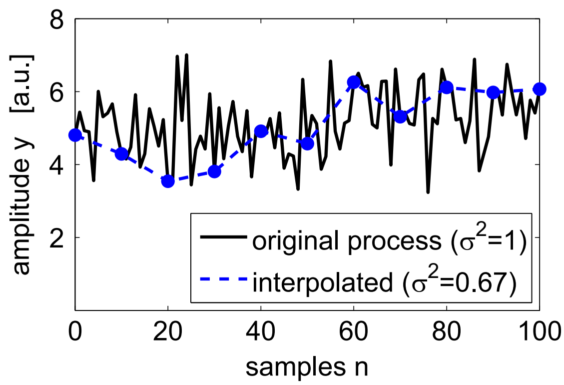

Fig. 1. Comparison of a generic highly unsteady process and the resulting data set after interpolation between discrete sample values.

Milovanovic, 2008). However, this will not solve the problem in principle, because these methods still use continuous functions, which lead to a strongly correlated result not adequate for highly fluctuating processes.

To overcome this limitation (Barnsley, 1986; Barnsley and Harrington, 1989) introduced generalized polynomial interpolation. This method is based on fractal functions and tailored for interpolating highly "wriggly" (Barnsley, 1986) functions, such as the elevation profiles in mountain ranges, stock-market indices, or the profile of cloud tops. However, the method lacks flexibility concerning conditions on the interpolation points and is mathematically rather involved (Bouboulis, 2012). Hence, even 30 years after its introduction fractal interpolation is not used widely in the engineering community (Navascués et al., 2014), while linear interpolation remains the default method.

To limit the scope of this paper, we focus on wind turbine engineering. Here, linear interpolation certainly is the most common strategy to obtain local blade inflow velocities from a turbulent wind field pre-computed from an industry standard spectrum. For example, the two major wind turbine simulation tools, *FAST* (Jonkman and Buhl, 2005) and *GH Bladed* (Bladed, 2012), employ piecewise linear interpolation to map from discrete wind speeds on a regular spatial grid to blade-local velocities. Based on Taylor's frozen turbulence hypothesis (see e.g. Panofsky and Dutton, 1984) these tools interpolate local apparent wind speeds linearly onto the rotating blades at each time step, while an a priori computed block of discrete frozen wind is stepped through the rotor disc. However, as just discussed in general (cf. Fig. 1), for a highly unsteady processes (with short correlation scales) such as turbulent wind, this approach can introduce a significant error into the statistical properties of the data set.

The wind velocity time record of turbulent wind in the atmospheric boundary layer can be interpreted as a stochastic field. Here the cross-correlation C is the indicator of what we called fluctuation length scale in the general case above. For the wind speed signals at any two points P_i and P_j the cross-correlation C is defined via the cross and auto spectrum of the two signals, S_{ij} and S_{ii}, respectively (Burton et al., 2011):

$$C(f, \Delta r) = \frac{|S_{ij}(f, \Delta r)|}{S_{ii}(f) S_{jj}(f)} \tag{1}$$

Obviously C is a function of the signal frequency component f as well as of the distance Δr between P_i and P_j. The commonly used wind turbine design standard IEC 61400-1, Edition 3 gives an empirical approximation equation for $C(f, \Delta r)$. The values decay quickly with increasing Δr, e.g. for 10 m/s wind speed $C(f = 1\ \text{Hz}, \Delta r = 2\ \text{m}) = 0.091$, and $C(f = 1\ \text{Hz}, \Delta r = 5\ \text{m}) = 0.024$. Hence, even for small distances linear interpolation between neighboring wind speeds means averaging two weakly correlated events and thus smoothing the data. The consequences are as discussed above (Fig. 1).

Veers (1988) was already aware of this loss of variance. Based on the cross-correlation function between the two support points of given data he derived an analytical expression for the resulting variance error. As a remedy he suggests without further details to add white noise to the interpolated data to recover the lost variance. Although this method can restore the desired variance it distorts the power spectrum by neglecting auto-correlation—an important characteristic for wind speed data and other physical processes.

A better method, which is based on stochastic increments and preserves both the signal's variance and spectrum, will be introduced in the next section. Rather than deriving yet another mathematically rigorous but practically too complicated interpolation theory, our goal was to devise a simple engineering method that provides a solution to the interpolation problem and an improvement over linear interpolation as currently used in wind turbine engineering, but without digging too deep into probabilistic math.

The resulting method will be presented for the one dimensional case first in general (Section 2.1), such that it can be easily transferred to any unsteady, weakly correlated/highly fluctuating (one dimensional) process in any field. Section 2.2 will provide a graphic application example: the method will be extended to higher dimensions and applied to a specific interpolation problem in wind turbine design. Results will be presented in Section 3, and compared against linear wind interpolation, the current status-quo, which is used as a baseline case here.

2. A new interpolation strategy: stochastic increment interpolation

While linearly interpolating (as well as interpolation based on continuous functions in general) does not always yield 'good' results, reducing the interpolation length down to the correlation length through finer spacing of known support points, or even obtaining the whole unsteady process at each required point from its fundamental statistical properties (e.g. probability density function, spectrum, spatial and temporal correlation, etc.) is often too tedious (Rai et al., 2015). For turbulent wind, for example, the computational effort for simulating the field rises with the fourth power of the number of grid points (Bladed, 2012). Moreover, the location of required inter-grid points is often not known a priori. Hence some kind of interpolation is inevitable. As an alternative to linear interpolation the use of data increments is suggested. This section will first outline the method in general. Subsequently we apply it to the specific example of wind turbine interpolation.