HOSTED BY

ELSEVIER

Available online at www.sciencedirect.com

ScienceDirect

journal homepage: www.elsevier.com/locate/jtte

Original Research Paper

Application of semi-analytical finite element method coupled with infinite element for analysis of asphalt pavement structural response

Pengfei Liu, Dawei Wang, Markus Oeser*

Faculty of Civil Engineering, RWTH Aachen University, Aachen D-52074, Germany

ARTICLE INFO

Article history:
Available online 20 January 2015

Keywords:
Asphalt pavement
Structural response
Semi-analytical finite element method
ABAQUS
Infinite element
Finite-infinite element coupling analysis

ABSTRACT

A specific computational program SAFEM was developed based on semi-analytical finite element (FE) method for analysis of asphalt pavement structural responses under static loads. The reliability and efficiency of this FE program was proved by comparison with the general commercial FE software ABAQUS. In order to further reduce the computational time without decrease of the accuracy, the infinite element was added to this program. The results of the finite-infinite element coupling analysis were compared with those of finite element analysis derived from the verified FE program. The study shows that finite-infinite element coupling analysis has higher reliability and efficiency.

1. Introduction

In the past decades, the finite element (FE) method has been developed rapidly and was increasingly used in many industrial fields as well as in the routine pavement design process. But there are several limitations in the conventional FE packages, such as the complexity of the program and hence the time-consuming user training process or over-simplifications of the modeling (Liu et al., 2014a). Therefore, it is necessary to find means that both improve the computational speed without increasing the resource requirement and keep the computational accuracy. One specified program SAFEM was developed based on the semi-analytical FE method to meet these requirements.

For a typical pavement structure problem as shown in Fig. 1, the geometry and material properties usually do not vary in the z-direction, but the boundary conditions, e.g. the load terms, exhibit a significant variation in that direction. Due to this characteristic, the pavement structure problem could not be simplified as a 2D plane strain case. An alternative method is to simplify the pavement structure as a 2D axisymmetric case, and the response under multiple loads can be obtained using superposition principle. However, the models based on the axisymmetric formulation cannot realistically model unidirectional loads, cracks or discontinuities within a pavement system and the ability to realistically model non-uniform contact pressures is also limited (Fritz, 2002). Zienkiewicz and Taylor (2005) proposed one method that assuming the displacements in the geometrical z-

* *Corresponding author*. Tel.: +49 241 8020389; fax: +49 241 80 22141.
E-mail address: liu@isac.rwth-aachen.de (P. Liu).

Peer review under responsibility of Periodical Offices of Chang'an University.
http://dx.doi.org/10.1016/j.jtte.2015.01.005

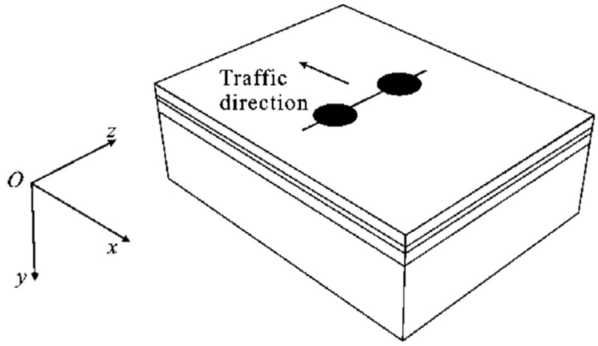

Fig. 1 – Pavement structure geometry and load mode (Liu et al., 2014a).

direction can be represented using Fourier series and exploiting its orthogonal properties, the problem of such a class can thus be simplified into a series of 2D solutions. This method is so called the semi-analytical FE method which has potential for overcoming the difficulties mentioned above.

The pavement length along the traffic direction and the thickness of sub-grade can be considered infinity. In order to minimize the influence of these boundaries, the FE-mesh must be sufficiently big in horizontal and vertical directions. This, however, means that a relatively large number of FE-elements are required to appropriately discretize the mesh. One of the methods that can keep the number of FE-elements reasonably low is the application of infinite elements at the infinite boundaries of the system in the FE method (Li et al., 2007b). This concept of the infinite element was firstly proposed by Ungless (1973), and then modified and further developed by many other researchers such as Beer and Meer (1981), Bettess (1977, 1980), Bettess and Zienkiewicz (1977), Zienkiewicz et al. (1983) and applied to a variety of problems (Hjelmstad et al., 1997; Li et al., 2007a; Wang and Brill, 2013). Currently, the infinite elements have been developed from one-dimensional (1D), unidirectional to 3D and multidirectional approaches. The concept of infinite elements can be generally divided into two categories: the mapping infinite element and decaying infinite element (Sallah and Buchanan, 1990). The features of the infinite elements can be concluded as follows (Jiang et al., 2009; Zhu and Bian, 2001):

- The mapping from finite field in the natural coordinate to infinite field in the global coordinate, e.g. when the natural coordinate ξ approaches 1 the corresponding global coordinate will trend towards infinity to fulfill the requirement that the computational field trends towards infinity.
- The decay of the displacement in the infinite field, e.g., when ξ approaches 1, the displacement tends towards 0 to fulfill the requirement of the boundary condition that the displacement at the infinity is 0.

In the following sections, the mathematical basis of the SAFEM and the 2D mapping infinite element will be presented at first, followed by a verification of the SAFEM by comparison with ABAQUS as well as a verification of the finite-infinite element coupling analysis in the SAFEM program. Finally, a brief summary and conclusions are provided at the end of this paper.

2. Description of semi-analytical finite element method

The first step in the FE formulation of SAFEM is to express the element coordinates and element displacements in the form of interpolation functions using the natural coordinate system of the element.

By using the SAFEM, the general form of the shape functions can be written as a Fourier series in which z ranges between 0 and a (Fritz, 2002; Hu et al., 2008; Liu et al., 2013, 2014a, 2014b; Zienkiewicz and Taylor, 2005), as shown in Fig. 2.

$$u = \sum_{k=1}^{6} N_k(x,y,z)u_k = \sum_{l=1}^{L}\sum_{k=1}^{6}\left\{\left[\overline{N_k}(x,y)\cos\frac{l\pi z}{a}\right] + \left[\overline{\overline{N_k}}(x,y)\sin\frac{l\pi z}{a}\right]\right\}u_k^l \tag{1}$$

where l identifies the term of the Fourier series, L is the total number of Fourier terms considered, $\overline{N_k}$ and $\overline{\overline{N_k}}$ are the shape functions of the element at node k.

The loading function defining the variation of load along the z-direction is given by Zienkiewicz and Taylor (2005).

$$f = \sum_{l=1}^{L}\left\{\left[\overline{p}(x,y)\cos\frac{l\pi z}{a}\right] + \left[\overline{\overline{p}}(x,y)\sin\frac{l\pi z}{a}\right]\right\} \tag{2}$$

where $\overline{p}(x,y)$ and $\overline{\overline{p}}(x,y)$ represent the pavement load.

The pavement is assumed to be supported on both side faces ($z = 0$ and $z = a$) in a manner preventing all displacements in the xy plane but permitting "unrestricted" motion in the z-direction. The Fourier series expansion should meet this requirement of the boundary condition. The displacement functions with three components u, v and w can be written as follow

$$U = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \sum_{l=1}^{L}\sum_{k=1}^{6} N_k \begin{bmatrix} \sin\frac{l\pi z}{a} & 0 & 0 \\ 0 & \sin\frac{l\pi z}{a} & 0 \\ 0 & 0 & \cos\frac{l\pi z}{a} \end{bmatrix} \begin{Bmatrix} u_k^l \\ v_k^l \\ w_k^l \end{Bmatrix} = \sum_{l=1}^{L} N^l \cdot U^l \tag{3}$$

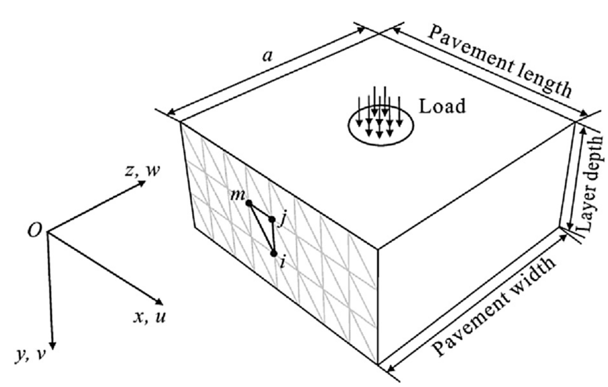

Fig. 2 – Schematic representation of a SAFEM situation (Liu et al., 2014a).