J. Wind Eng. Ind. Aerodyn. 151 (2016) 48–59

Contents lists available at ScienceDirect

Journal of Wind Engineering and Industrial Aerodynamics

journal homepage: www.elsevier.com/locate/jweia

An efficient simulation method for vertically distributed stochastic wind velocity field based on approximate piecewise wind spectrum

Koffi Togbenou [a], Yongle Li [a,*], Ning Chen [a,b], Haili Liao [a]

[a] Department of Bridge Engineering, Southwest Jiaotong University, 610031 Chengdu, Sichuan, China
[b] School of Civil Engineering, Hunan University of Science and Technology, 411201 Xiangtan, China

ARTICLE INFO

Article history:
Received 30 March 2015
Received in revised form
29 December 2015
Accepted 5 January 2016
Available online 10 February 2016

Keywords:
Stochastic wind velocity
Approximate wind spectrum
Cholesky decomposition
Kaimal spectrum

ABSTRACT

Stochastic wind velocity simulation based on the spectral representation method often requires considerable computation time due to the repetitive Cholesky decomposition of the power spectral density matrix. In this paper, a modified model is proposed for the Davenport coherence function and is used to improve the efficiency of the Cholesky decomposition based on an approximation technique where the target wind spectrum is reformulated as a piecewise function. Based on this new formulation where the major piece of the spectrum is regarded as a product of separate functions of frequency and height, vertically distributed wind velocity field can be efficiently generated through a closed-form expression of the Cholesky decomposition. When the target spectrum is not separable the Cholesky decomposition can still be avoided based on the proposed coherence function. In other words, the proposed simulation method does not require either the direct Cholesky decomposition or the eigenvector decomposition. Numerical investigations show that, more than 60% of computation time can be saved for the decomposition of the power spectral density matrix involved in the simulation of two hundred wind velocity processes and the entire simulation procedure is 3 times faster than the ordinary spectral representation method for the simulation of two hundred processes.

1. Introduction

Civil engineering structures such as long span bridges, high-rise buildings and towers are very sensitive to wind action. They demonstrate wind-induced vibration which is usually studied during their aerodynamic analysis. The most important element in the aerodynamic analysis of a given structure is the velocity time-history of the stochastic wind around the structure. Currently, there are several methods available to generate the stochastic wind velocity time-history. Most of them are based on Monte Carlo simulation (Spanos and Zeldin, 1998; Kareem, 2008) which can be achieved by frequency domain approaches such as linear filtering approaches (Li and Kareem, 1990; Deodatis and Shinozuka, 1988; Spanos and Mignolet, 1989) or by spectral representation method (Chen and Letchford, 2004; Li and Kareem, 1991; Shinozuka and Jan, 1972; Grigoriu, 1993). Compared to the linear filtering approaches, the spectral representation method (SRM) is more simple and accurate; therefore it is widely used. However, the spectral representation method is found to be time consuming due to the repetitive Cholesky decomposition of the power spectral density (PSD) matrix. This issue has been addressed in the past decades and several techniques have been developed to improve the efficiency of the decomposition.

An alternative to the Cholesky decomposition is the eigenvector decomposition also referred to as the proper orthogonal decomposition (POD). The POD approach (Holmes et al., 1997; Rathinam and Petzold, 2003; Di Paola and Gullo, 2001; Solari and Carassale, 2000; Chen and Kareem, 2005) is widely used to reduce variables in large scale system based on the truncation of the higher eigenmodes. The approach can save considerable computation time and can also be applied for the simulation of non-Gaussian and non-stationary processes (Phool et al., 2002, 2005; Huang et al., 2001). However, the truncation of the higher eigenmodes may lead to unacceptable underestimation of local wind load (Chen and Kareem, 2005).

Ding et al. (2006), Gao et al. (2012), and Carassale and Solari, (2006) utilized some interpolation techniques to increase the efficiency of the Cholesky decomposition. For long span cable-stayed bridge, Li et al. (2004) introduced a simplification approach to improve the computational complexity of the decomposition by treating a three-dimensional wind velocity field as a group of one dimensional wind velocity fields. Recently, Huang et al. (2013) developed a new approach for the Cholesky decomposition of

* Corresponding author. Tel.: +86 28 87601119.
E-mail addresses: florest2@yahoo.fr (K. Togbenou), lele@swjtu.edu.cn (Y. Li), chen-ning-520@163.com (N. Chen), hlliao@swjtu.edu.cn (H. Liao).

http://dx.doi.org/10.1016/j.jweia.2016.01.005
0167-6105/

complex power spectral density matrix. The approach consists of separating the phases from the complex PSD matrix. Significant computation time can be saved by decomposing only the derived real modulus matrix. However, the approach is limited to complex PSD matrix.

A closed-form expression of the Cholesky decomposition was established by Yang et al. (1997) and was later improved by Cao et al. (2000) for the simulation of wind velocity field on bridge deck based on the assumption that the simulation points are uniformly distributed at the same height along the bridge deck axis. The approach has remarkably increased the simulation efficiency since the direct Cholesky decomposition is not required. The approach is well known for its higher efficiency and is widely used to generate horizontally distributed wind velocity field. Unfortunately, lots of the structures encountered in civil engineering (such as bridge towers, high-rise buildings, chimneys and so on) are vertical with varying heights. The closed-form expression of the Cholesky decomposition as presented in (Yang et al., 1997) cannot be directly applied to these structures since the wind field is vertically distributed along these structures and the wind spectrum varies with height. This limitation is the main issue addressed in this paper for vertically distributed wind velocity field.

In this paper, a modified model of the Davenport coherence function is suggested and used to improve the efficiency of the Cholesky decomposition of the power spectral density matrix for the simulation of vertically distributed wind velocity field based on an approximation technique where the target wind spectrum is reformulated as a piecewise function in which the major piece of the spectrum is regarded as a product of separate functions of frequency and height. The Cholesky decomposition of the power spectral density matrix involved in the simulation of wind velocity fluctuation along one dimensional height varying structures can be then explicitly expressed using the approximate spectrum. The spectrum separation idea is developed specially according to Kaimal spectrum but can also be applied to the most frequently used spectra such as the models of Simiu, Davenport and Harris. When the selected target spectrum is not separable, the closed-form expression of the Cholesky decomposition can still be derived based on the proposed coherence function but with additional computations required. The proposed method is very efficient since it requires neither the direct Cholesky decomposition nor the eigenvector decomposition. Both the accuracy and the efficiency of the proposed simulation method are demonstrated through a numerical example involving the simulation of fluctuating wind velocity field along a vertical bridge tower.

2. Simulation of stochastic wind velocity field by spectral representation method

Let assume that $P(x,y,z)$ is a point of the Cartesian coordinate system where the x-axis represents the along-wind direction, the y-axis represents the lateral wind direction and z is the height above ground. The wind velocity fluctuation at $P(x,y,z)$ can be regarded as a time-dependent $1V-3D$ random field process. For structure with n simulation points $[P_1(x,y,z),\ldots,P_n(x,y,z)]$, the wind velocity fluctuation acting on the structure can be represented as a time-dependent $nV-3D$ random field. The correlation among the three fluctuation components in x, y and z directions can be assumed to be weak (Simiu and Scanlan, 1986) so that each of them can be represented as a time-dependent $nV-1D$ stochastic field which is usually assumed to be stationary Gaussian.

Let us denote $S^0(\omega)$ the two-sided cross-spectral density matrix of a one-dimensional multivariate stationary Gaussian process:

$$S^0(\omega)=\begin{bmatrix} S^0_{11}(\omega) & S^0_{12}(\omega) & \cdots & S^0_{1n}(\omega) \\ S^0_{21}(\omega) & S^0_{22}(\omega) & \cdots & S^0_{2n}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ S^0_{n1}(\omega) & S^0_{n2}(\omega) & \cdots & S^0_{nn}(\omega) \end{bmatrix} \tag{1}$$

$$S^0_{jk}(\omega)=\sqrt{S^0_{jj}(\omega)S^0_{kk}(\omega)}\,\Gamma_{jk}(\omega) \tag{2}$$

where $S^0_{jj}(\omega)$ and $S^0_{kk}(\omega)$ are the auto spectral density functions of the components $x_j(t)$ and $x_k(t)$, respectively; $\Gamma_{jk}(\omega)$ is the coherence function. The cross-spectral density matrix $S^0(\omega)$ is hermitian and can be decomposed in the following form:

$$S^0(\omega)=H(\omega)^{T^*}H(\omega) \tag{3}$$

$$H(\omega)=\begin{bmatrix} H_{11}(\omega) & 0 & \cdots & 0 \\ H_{21}(\omega) & H_{22}(\omega) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ H_{n1}(\omega) & H_{n2}(\omega) & \cdots & H_{nn}(\omega) \end{bmatrix} \tag{4}$$

where the superscripts T and * denote matrix transpose and complex conjugate, respectively. $H(\omega)$ is a lower triangular matrix and can be computed using the Cholesky decomposition. In general, the off-diagonal elements of $H(\omega)$ are complex functions of ω and can be expressed as follows:

$$H_{jk}(\omega)=\left|H_{jk}(\omega)\right|e^{i\theta_{jk}(\omega)} \tag{5}$$

where $\theta_{jk}(\omega)$ is the angle given by

$$\theta_{jk}(\omega)=\tan^{-1}\left(\frac{\mathrm{Im}\left[H_{jk}(\omega)\right]}{\mathrm{Re}\left[H_{jk}(\omega)\right]}\right) \tag{6}$$

For real-valued coherence function, all of the elements of the matrix $H(\omega)$ are real and the angle $\theta_{jk}(\omega)$ is equal to zero for any value of ω. That is:

$$H_{jk}(\omega)=H^*_{jk}(\omega) \tag{7}$$

$$\theta_{jk}(\omega)=0 \tag{8}$$

The wind velocity field can be simulated by the following formula (Deodatis, 1996a, 1996b):

$$x_j(t)=2\sqrt{\Delta\omega}\sum_{l=1}^{N}\sum_{m=1}^{j}\left|H_{jm}(\omega_{ml})\right|\cos\left(\omega_{ml}t+\varphi_{ml}\right) \tag{9}$$

where $j=1,2,\cdots,n$; $\omega_{ml}=(l-1)\Delta\omega+\dfrac{m}{n}\Delta\omega$; $m=1,2,\cdots,n$; $l=1,2,\cdots,N$; $\Delta\omega=\frac{\omega_u}{N}$

ω_u is the upper cutoff frequency beyond which $S^0_{jk}(\omega)$ may be assumed to be zero. In Eq. (9), $\varphi_{1l},\varphi_{2l},\cdots,\varphi_{nl}$, $l=1,2,\cdots,N$ are n sequences of independent random phase angles uniformly distributed over the interval $[0,2\pi]$.

Alternatively, the power spectral density matrix can be expressed as (Gao et al., 2012; Li et al., 2004)

$$S^0(\omega)=D(\omega)\Gamma(\omega)D^T(\omega) \tag{10}$$

where

$$D(\omega)=\mathrm{diag}\left[\sqrt{S^0_{11}(\omega)},\sqrt{S^0_{22}(\omega)},\ldots,\sqrt{S^0_{nn}(\omega)}\right] \tag{11}$$

$\Gamma(\omega)$ is a positive definite Hermitian matrix. Therefore, $S^0(\omega)$ can be factorized in the following form:

$$S^0(\omega)=D(\omega)L(\omega)L^T(\omega)D^T(\omega)=H(\omega)^TH(\omega) \tag{12}$$