J. Wind Eng. Ind. Aerodyn. 99 (2011) 748–756

Contents lists available at ScienceDirect

Journal of Wind Engineering and Industrial Aerodynamics

journal homepage: www.elsevier.com/locate/jweia

Critical conditions of galloping for inclined square cylinders

Giuseppe Piccardo *, Luigi Carassale, Andrea Freda

Department of Civil, Environmental and Architectural Engineering (DICAT), University of Genoa, Via Montallegro 1, 16145 Genoa, Italy

A R T I C L E I N F O

Available online 30 April 2011

Keywords:
Aeroelastic phenomena
Yawed galloping
Critical conditions
Dynamic instability
Fluid–structure interaction

A B S T R A C T

This paper concerns the study of galloping critical conditions on square cylinders with an arbitrary attitude in the wind flow. It is based on a consistent theoretical model of wind actions, which permits the analysis of a generic-shaped cross-section. A generalized definition of the aerodynamic damping matrix as function of both the angle of incidence and the angle of skew is proposed. The availability of specific experimental data allows a sound definition of aerodynamic actions. The analysis of critical conditions can be performed in closed form in the plane of the aerodynamic damping matrix invariants. Preliminary results obtained in smooth-flow conditions point out that yaw effects might influence critical conditions from a quantitative point of view, but do not destroy the instability domain obtained in the classical cross-flow conditions.

1. Introduction

Galloping is an aeroelastic phenomenon related to slender structures with non-circular cross-section; it appears with large-amplitude oscillations, primarily in the cross-wind direction. Galloping has mainly been studied under cross-flow conditions; in the literature few contributions concern the effect of the skew angle on galloping instability both from a theoretical and an experimental point of view (e.g., Skarecky, 1975; Shum et al., 2009). Besides, their aim is mainly devoted to analyze the amplitude of vibration in the non-linear post-critical range. Nevertheless, from a technical point of view, the knowledge of critical conditions is relevant even if the structure could be compatible with large-amplitude non-linear oscillations, as in the case of suspended cables. Objective problems that have made hard the development of this topic are the complexity of the flow pattern past the body, the difficulty of modeling the associated forces, and the lack of experimental data in yawed configurations. The interest about this subject is considerably increased over the last few years owing to the fact that galloping-like instability phenomena have been observed in yawed conditions also on dry, smooth circular cylinders (see, e.g., Carassale et al., 2005; Cheng et al., 2008a).

This paper concerns the study of galloping critical conditions on square cylinders with an arbitrary attitude in the wind flow. It is based on a consistent theoretical model of wind actions (Freda, 2005), which enables the analysis of a generic-shaped cross-section. The availability of specific experimental results (Freda and Tamura, in preparation) allows a sound definition of aerodynamic actions. The analysis of critical wind velocities can be performed in closed form in the plane of the aerodynamic damping matrix invariants, neglecting torque actions and considering internal resonance conditions, through a suitable non-dimensional form of the equations of motion (Luongo and Piccardo, 2005). The objective of this work is to investigate the role of the skew angle in the evaluation of galloping onset, and to propose non-dimensional solutions valid for any squared elements, independently from their size and mechanical characteristics.

2. Theoretical model of wind forces

Let us consider, at first, a slender, rigid fixed cylinder with a generic cross-section (Fig. 1a), with a local reference system described by the unit vectors $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$. The unit vector $\mathbf{n}$, representing the direction of the mean wind velocity, is expressed as

$$\mathbf{n} = \frac{\mathbf{U}}{\|\mathbf{U}\|} = \begin{bmatrix} \cos\alpha\cos\beta & \sin\alpha\cos\beta & \sin\beta \end{bmatrix}^T \quad (1)$$

where $\|\mathbf{U}\| = U$ is the Euclidean norm of the wind velocity vector $\mathbf{U}$. The angles α and β describe, respectively, the rotation in the A-plane around $\mathbf{e}_3$ direction (i.e., the classical angle of incidence or angle of attack, Fig. 1b), and the rotation in the B-plane around the projection of the lift direction on the A-plane (i.e., the skew angle, Fig. 1c). The aerodynamic force per unit length acting on the cylinder can be expressed as (Freda, 2005)

$$\mathbf{f} = \tfrac{1}{2}\rho b \|\mathbf{U}\|^2 \left[C_D \mathbf{d} + C_L \mathbf{l} \right] \quad (2)$$

where ρ is the air density, b is a reference size of the element, and C_D and C_L are, respectively, the drag and lift force coefficients to be evaluated experimentally measuring aerodynamic actions on a fixed

* Corresponding author. Tel.: +39 010 353 2970; fax: +39 010 353 2292.
E-mail addresses: giuseppe.piccardo@unige.it (G. Piccardo), luigi.carassale@unige.it (L. Carassale), andrea.freda@unige.it (A. Freda).

0167-6105/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.jweia.2011.03.009

Nomenclature

b	element reference size
$\mathbf{C}_a$	aerodynamic damping matrix
$\mathbf{C}_s$	mechanical damping matrix
C_D	pseudo-drag force coefficient
C_L	lift force coefficient
$\mathbf{D}$	total damping matrix
$\mathbf{d}$	unit vector of the pseudo-drag direction
$\mathbf{e}_k$	unit vectors of the local reference system
$\mathbf{f}$	aerodynamic force per unit length
$\mathbf{K}$	stiffness matrix
$\mathbf{l}$	unit vector of the lift direction
$\mathbf{M}$	mass matrix
m	element mass per unit length
$\mathbf{n}$	mean wind speed direction
q_k, $\tilde{q}_k$	dimensional and non-dimensional components of the cylinder velocities, respectively
t, $\tilde{t}$	dimensional and non-dimensional time, respectively
$\mathbf{U}$	wind velocity vector
$\mathbf{z}$	non-dimensional instantaneous fluctuation of the wind-element relative velocity
A	plane of the element cross-section
α	rotation in the A-plane around $\mathbf{e}_3$ (angle of incidence)
B	plane of the wind speed and of the element axis
β	rotation in the B-plane around the projection of the lift direction on A (skew angle)
μ	dimensionless wind speed
μ_{cr}	dimensionless critical wind speed (two degree-of-freedom)
μ_{gDH}	dimensionless critical wind speed (generalized Den Hartog criterion)
ξ, ξ_k	mechanical damping ratios
ρ	air density
ω	ratio between structural natural frequencies
ω_k	structural natural frequencies
$*$	instantaneous quantities
$'$	first derivative with respect to the specified variable

cylinder by appropriate wind tunnel tests; in general, they depend on the wind direction $\mathbf{n}$ (i.e., on the angles α and β) as well as on the Reynolds number.

It should also be noted that the drag force is usually defined as the force acting in the direction of the mean wind velocity; in the case analyzed here, it seems appropriate to mention a pseudo-drag force since it is defined in the direction of the mean wind velocity projected on the A-plane, where the element cross-section lies. Thus, $\mathbf{d}$ and $\mathbf{l}$ are the unit vectors of the pseudo-drag and lift forces, laying on the A-plane and rotated α from $\mathbf{e}_1$ and $\mathbf{e}_2$, respectively (Fig. 1b); they can also be conveniently expressed as

$$\mathbf{d} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \end{bmatrix}^T = \frac{1}{\cos\beta}\begin{bmatrix} n_1 & n_2 & 0 \end{bmatrix}^T$$
$$\mathbf{l} = \begin{bmatrix} -\sin\alpha & \cos\alpha & 0 \end{bmatrix}^T = \frac{1}{\cos\beta}\begin{bmatrix} -n_2 & n_1 & 0 \end{bmatrix}^T \quad (3)$$

where n_1 and n_2 are the components of the unit vector $\mathbf{n}$ along $\mathbf{e}_1$ and $\mathbf{e}_2$, respectively.

In expression (2) torsional effects are implicitly neglected: they are generally present (unless particular cases of symmetry

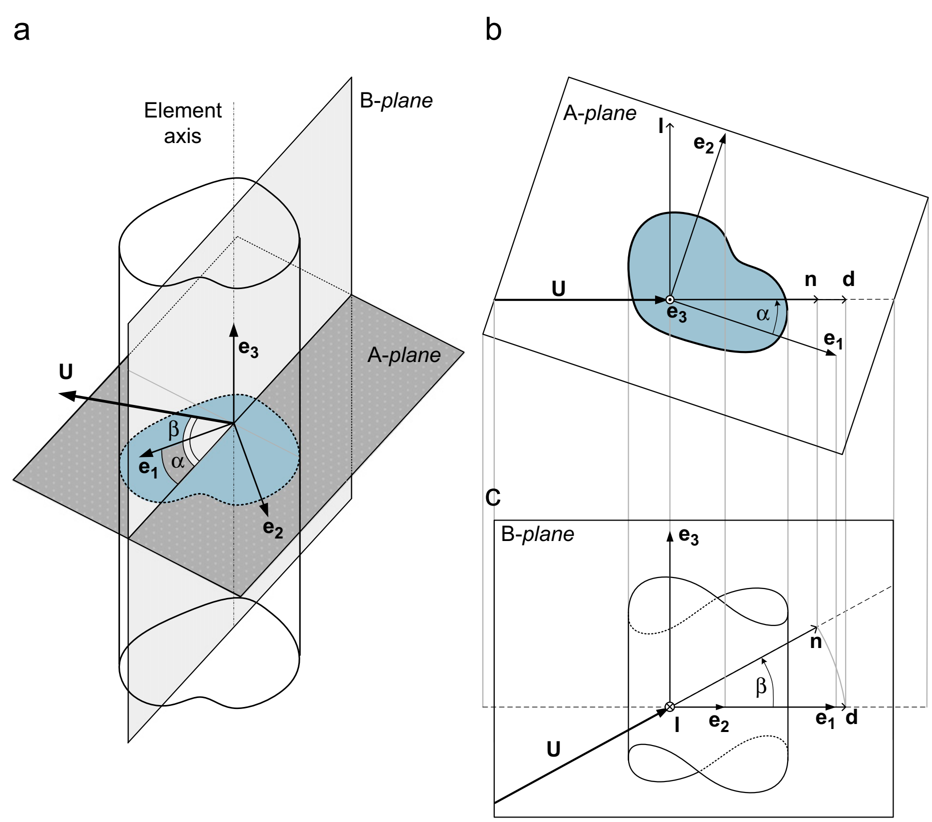

Fig. 1. Fixed element immersed in a skewed flow: (a) 3D view, (b) cross-sectional view (A-plane), and (c) lateral view (B-plane).