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Revisiting combination rules for estimating extremes of linearly combined correlated wind load effects

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A R T I C L E I N F O

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A B S T R A C T

This study re-evaluates the performance of some approximate combination rules used in practice for estimating the mean extremes of linearly combined correlated wind load effects. The Turkstra's rule and its variants are focused, which are also often referred to as the "coincident action" or "companion action" methods and have been widely applied to the combinations of wind loads and responses as well as other dynamic responses. The probability distributions of estimations from the Turkstra's rule and its variants are derived, which permit the assessment of their performance for various correlated responses through analytical formulations rather than response time history samples. The analytical formulations and the performance of the Turkstra's rule and its variants are also validated using simulated response time histories of a high-rise building with coupled three-dimensional mode shapes. This study reemphasizes the difference in the correlation coefficients of wind loads and wind-excited responses, and highlights the importance of response correlation coefficient (both the value and sign) for the estimations of extremes of resultant response and its absolute value. The results illustrate that the approximate combination rules can considerably over- or underestimate the extremes of combined responses depending on the ratio and correlation coefficient of the response components. The results of this study help in better understanding the effectiveness and limitations of the approximate combination rules used in current practice.

1. Introduction

Assessment of building performance to strong winds requires estimations of multiple important responses, which can be given as linear or nonlinear combinations of response components in principle directions in terms of alongwind, crosswind and torsional responses. One example of the linear response combinations is the bending normal stress of a column member which is the sum of the stresses caused by bending moments in two translational principle directions. One example of nonlinear response combinations is the vectorial summation of building accelerations where the magnitude of acceleration at a given building floor location is arrived at as square-root-of-sum-of-square (SRSS) of accelerations in two translational directions. In engineering practice, it is often desired to estimate the extremes (peaks) of these resultant responses directly from the extremes of response components according to some combination rules.

In the case of linear combinations, the standard deviation (STD) of a resultant response is accurately calculated from the STDs of the response components using complete quadratic combination (CQC) rule (Der Kiureghian, 1980), where the correlation coefficient of response components plays an important role and needs to be adequately quantified. The CQC rule reduces to SRSS rule when the response components are uncorrelated. The CQC rule can be approximately extended to the estimation of mean extreme of resultant response from the mean extremes of response components under the presumption that the peak factors for the response components and resultant response are not very different. This assumption is generally acceptable when responses are Gaussian processes, while it can result in a considerable error when response processes are non-Gaussian (Gong and Chen, 2014).

Although the CQC rule is sufficiently accurate for linearly combined responses, simplified combination rules are often used in design codes and standards. For instance, the 40% and 75% rules are often used to replace the SRSS rule. The Turkstra's rule (Turkstra, 1970) and its modified or extended versions (variants) have also been widely used in practice where the extreme of one response component and the simultaneous values of other components at the point-in-time are used for combination. These rules are also often referred to as the "coincident action" or "companion action" methods. These rules are under the assumption that the extreme of resultant response occurs precisely when one of the component processes takes its extreme value. The original

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Turkstra's rule was intended for the combination of independent load effects. The underestimation of the Turkstra's rule has been pointed out in the literature (e.g., Naess and Røyset, 2000). Naess and Røyset (2000) examined two variants of Turkstra's rule for combinations of dependent load effects. A modified combination scheme using multiple points-in-time based on response time history samples was suggested in Yeo (2013), while it sacrifices the advantage of using simplified combination rules such as the Turkstra's rule.

A number of studies concerning the linear combinations of alongwind, crosswind and even torsional wind load effects have been reported in the literature (Melbourne, 1975; Vickery and Basu, 1984; Solari and Pagnini, 1999; Tamura et al., 2000, 2001, 2003, 2008; Bartoli et al., 2011; Tamura et al., 2014). Tamura et al. (2000, 2001, 2003, 2008) carried out comprehensive investigations on the combinations of the maximum value of one of the wind force components with two other simultaneously recorded force components for low- and middle-rise buildings using pressure measurement data. A similar study on the wind load combinations following the basic idea of Turkstra's rule has also been extended to high-rise buildings (Tamura et al., 2014). Bartoli et al. (2011) discussed the quasi-static combination of wind loads using a copula-based approach for modeling the joint probability distributions of extremes of load components. ASCE 7-10 (2010) presents relative simple load combinations for buildings, where 75% of alongwind load are simultaneously applied in both alongwind and acrosswind directions, while torsional load can also be included when there are eccentricities. Two different procedures are presented in AIJ-RLB-2004 (AIJ, 2004): one for low- and middle-rise buildings, and another for high-rise buildings (Tamura et al., 2014; Asami, 2000). In some practice (SOM, 2004), the response in one primary direction with a target mean recurrence interval (MRI) was combined with a response in another direction having a reduced MRI, while the actual MRI of such a combination is not clear.

This study re-evaluates the performance of some approximate combination rules used in practice for estimating the mean extremes of linearly combined resultant wind load effects. The probability distributions of estimations from the Turkstra's rule and its variants are derived, which permit the assessment of their performance for various correlated responses through analytical formulations rather than response time history samples. The analytical formulations and the performance of the Turkstra's rule and its variants are also validated from time domain response simulations of a high-rise building with coupled 3D mode shapes. This study reemphasizes the difference in the correlation coefficients of wind loads and wind-excited responses, and highlights the importance of response correlation coefficient (both the value and sign) for the estimations of extremes of resultant response and its absolute value. The results of this study illustrate the effectiveness and limitations of the approximate combination rules used in current practice.

2. Extreme value distribution and peak factor of a Gaussian process

The cumulative distribution function (CDF) of extreme value of a zero-mean Gaussian process $R(t)$ over a time duration T is given as follows under the Poisson assumption of crossings:

$$F_{r\max}(r) = \Pr(R_{\max} \le r) = \exp\left\{ -\nu_0 T \exp\left[-\frac{r^2}{2\sigma_r^2} \right] \right\} \tag{1}$$

where $\nu_0 = \sigma_{\dot r}/(2\pi\sigma_r)$ is the mean upcrossing rate at the zero-mean level; σ_r and $\sigma_{\dot r}$ are the STDs or root-mean-square (RMS) values of $R(t)$ and $\dot R(t) = dR(t)/dt$, respectively; r is the response level; and T is the duration of time.

The p-fractile value of the extreme, i.e., $F_{r\max}(r_{p\max}) = p$, is then calculated as

$$r_{p\max}/\sigma_r = \sqrt{2\ln[\nu_0 T/\ln(1/p)]} \tag{2}$$

When the extreme value distribution is approximated as a Type I Gumbel distribution, the mean extreme corresponds to $p = 57\%$. The peak factor is then calculated as $g_r = r_{p\max}/\sigma_r$ with $p = 57\%$.

Davenport (1964) developed the following closed-form formulation for the peak factor:

$$g_r = \sqrt{2\ln(\nu_0 T)} + \frac{0.5772}{\sqrt{2\ln(\nu_0 T)}} \tag{3}$$

Vanmarcke (1972, 1975) introduced an improved model for very narrow band processes by taking into account the crossing clustering. The CDF of the extreme is given as

$$F_{r\max}(r) = \Pr(R_{\max} \le r) = \exp\left\{ -\nu_0 T \exp\left[-\frac{r^2}{2\sigma_r^2} \right] \phi(r) \right\} \tag{4}$$

$$\phi(r) = \frac{[1 - \exp(-\sqrt{2\pi}\delta^{1.2} r/\sigma_r)]}{[1 - \exp(-r^2/2\sigma_r^2)]} \tag{5}$$

and the p-fractile value of extreme is approximated as

$$r_{p\max}/\sigma_r = \sqrt{2\ln\{r_p[1 - \exp(-\sqrt{2\pi}\delta^{1.2}\sqrt{2\ln r_p})]\}} \tag{6}$$

where $r_p = \nu_0 T/\ln(1/p)$, δ is the bandwidth parameter and is defined as

$$\delta = \sqrt{1 - \lambda_1^2/(\lambda_0\lambda_2)} \tag{7}$$

and λ_n is n-th moment of the process power spectral density (PSD) function $S_r(f)$:

$$\lambda_n = \int_0^\infty (2\pi f)^n S_r(f)\, df \tag{8}$$

When the extreme value of $|R(t)|$ is concerned, ν_0 and $\sqrt{2\pi}\delta^{1.2}$ in the proceeding formulations related to the single-barrier crossing, i.e., B-crossing, should be replaced by $2\nu_0$ and $\sqrt{\pi/2}\delta^{1.2}$, respectively, for the double-barrier crossing, i.e., D-crossing.

3. Complete quadratic combination (CQC) rule

In the following, the resultant response from a linear combination of two dynamic responses is considered, while the discussion can be readily extended into combinations of more than two response components. Any linear combination of two responses can be represented as a sum of two responses without loss of generality. The notation used in this study is based on using capital letters for random variables or processes and lower case letters for deterministic quantities or "dummy" values of these random terms. Consider the resultant response $R(t)$ given as a sum of $R_1(t)$ and $R_2(t)$ as

$$R(t) = R_1(t) + R_2(t) \tag{9}$$

The STD or RMS value of $R(t)$ is calculated as

$$\sigma_r = \left(\sigma_{r_1}^2 + \sigma_{r_2}^2 + 2\rho_{12}\sigma_{r_1}\sigma_{r_2}\right)^{1/2} \tag{10}$$

and the mean extreme of $R(t)$ is accordingly determined as

$$r_{\max 0} = g_r\sigma_r \approx \left(r_{1\max 0}^2 + r_{2\max 0}^2 + 2\rho_{12} r_{1\max 0} r_{2\max 0}\right)^{1/2} \tag{11}$$

where σ_{r_1} and σ_{r_2} are the RMS values of $R_1(t)$ and $R_2(t)$, respectively; and ρ_{12} is the correlation coefficient between $R_1(t)$ and $R_2(t)$; $r_{\max 0} = g_r\sigma_r$, $r_{1\max 0} = g_{r_1}\sigma_{r_1}$ and $r_{2\max 0} = g_{r_2}\sigma_{r_2}$ are the mean