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# Identification of flutter derivatives of bridge decks with free vibration technique

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ABSTRACT

The flutter derivatives of bridge decks can be determined in a unique manner on condition that the complex modal parameters of the system at one reduced frequency are obtained. Based on the idea, a new method of identifying the flutter derivatives of bridge decks is proposed and it can overcome some shortcoming of the existing method and extend the applicability of the free vibration technique at high wind velocity. The identified results have agreements with the target ones of an ideal thin-plate section and those of a thin-plate section measured by the forced vibration technique. The proposed method is reliable and effective to extract the flutter derivatives from coupled free vibration.



## 1. Introduction

The flutter derivatives of bridge decks are essential parameters to analyze the flutter stability of long-span bridges, which can be determined through wind tunnel tests. There are two kinds of experimental methods to obtain the parameters with the sectional model of bridge decks: the free vibration technique and forced vibration technique. The free vibration technique of the 2Degree-of-Freedom (DOF) sectional model is widely used due to its simplicity and availability, which is discussed in this paper.

At first, Scanlan et al. (1971) proposed a method of extracting the flutter derivatives of bridge decks with the free vibration technique. The method consists of two stages: firstly, uncoupled terms are obtained separately from pure vertical and torsional free oscillations; and secondly, coupled terms are obtained from coupled oscillations where the vertical and torsional motions of the model must have the same frequency at each wind velocity. Xie (1986) presented a coupled oscillation to identify the parameters of binary unsteady aerodynamic forces with the Kalman filter method, which can simplify the experimental procedure to some extent. Later on, coupled vibration approaches are widely employed to identify the flutter derivatives directly.

Yamada et al. (1992) introduced the extended Kalman filter (EKWGI) method into the identification procedure of the flutter derivatives from coupled oscillation data. In their work, wind tunnel tests were made at high wind speeds near the flutter onset speed and thus only the flutter mode could be practically excited during the free vibration. Sarkar et al. (1994) developed a modified Ibrahim time-domain (MITD) method to extract all the direct and cross derivatives from the coupled free vibration data of 2DOF model. This method requires selection of the time shift N1 and N2. They found a way to select these two time shifts close to optimal values. Furthermore, Iwanmoto and Fujino (1995) presented an identification method to obtain all eight derivatives simultaneously from free vibration data consisting of two modes and the uniqueness problem in reducing the identified quantities to non-dimensionalized flutter derivatives is discussed and an approximate model of the aerodynamic forces is proposed. The results of a flat box girder section by the presented method were compared with those measured by the forced vibration technique and an agreement between them in the range of low reduced velocity was found. Moreover, Gu et al. (2000) proposed a unifying least-square method to identify the flutter derivatives of bridge decks and Ding et al. (2001) modified the method through approving its stability and accuracy. Chowdhury and Sarkar (2003) developed a new system identification method (iterative least-squares method or ILS method) to efficiently extract the flutter derivatives using a section model suspended by a three-DOF elastic suspension system.

There are three main shortcomings involved in the existing method. Firstly, since the free vibration motion generally contains two modes (i.e. two frequencies), the 16 flutter derivatives corresponding to the two reduced velocities cannot be fully determined from the free vibration data at one wind velocity theoretically. Secondly, although the identified quantities are reduced to flutter derivatives of approximate aerodynamic forces

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in the existing method, it is difficult to determine the corresponding relations between these derivatives and the two reduced velocities. Lastly, at high wind velocity the flutter derivatives cannot be identified accurately because the aerodynamic damping of the vertical-dominant mode is too high to obtain vibration data enough for the identification.

It has been noted that the parameters of two pairs of complex modes of the system at one reduced frequency can be given using the characteristic analysis. On the contrary, the flutter derivatives of bridge decks can be determined in a unique manner on condition that the complex modal parameters of the system at one reduced frequency are obtained. Based on the idea, a new method of identifying the flutter derivatives of bridge decks is proposed in this paper and it can overcome some shortcoming of the existing method and extend the applicability of the free vibration technique at high wind velocity.

## 2. Identification of flutter derivatives

According to the flutter theory developed by Scanlan et al. (1971), the motion equations of a 2DOF bridge deck section in smooth flow can be written as

$$m(\ddot{h}+2\xi_h\omega_h\dot{h}+\omega_h^2 h)=L_{se} \tag{1a}$$

$$I(\ddot{\alpha}+2\xi_\alpha\omega_\alpha\dot{\alpha}+\omega_\alpha^2\alpha)=M_{se} \tag{1b}$$

where $m$ and $I$ are the model mass and mass inertia moment per unit length, respectively; $h$ and $\alpha$ are the verical and torsional displacement; $\xi_h$, $\omega_h$, $\xi_\alpha$ and $\omega_\alpha$ are the damping ratios and natural circular frequencies of vertical and torsional motions, respectively; $L_{se}$ and $M_{se}$ are the aerodynamic self-excited force and moment, respectively, given by

$$L_{se}=\frac{1}{2}\rho U^2(2B)\left[KH_1^*(K)\frac{\dot{h}}{U}+KH_2^*(K)\frac{B\dot{\alpha}}{U}+K^2H_3^*(K)\alpha+K^2H_4^*(K)\frac{h}{B}\right] \tag{2a}$$

$$M_{se}=\frac{1}{2}\rho U^2(2B^2)\left[KA_1^*(K)\frac{\dot{h}}{U}+KA_2^*(K)\frac{B\dot{\alpha}}{U}+K^2A_3^*(K)\alpha+K^2A_4^*(K)\frac{h}{B}\right] \tag{2b}$$

where $\rho$ is the air density; $U$ the mean wind velocity; $B$ the width of the deck model; $K$ $(=\omega B/U)$ the reduced frequency; $H_i^*$ and $A_i^*$ (i=1,2,3,4) are the flutter derivatives.

Denoting

$$H_1=\frac{\rho B^2}{m}H_1^*(K),\ H_2=\frac{\rho B^3}{m}H_2^*(K),\ H_3=\frac{\rho B^3}{m}H_3^*(K),\ H_4=\frac{\rho B^2}{m}H_4^*(K) \tag{3a}$$

$$A_1=\frac{\rho B^3}{I}A_1^*(K),\ A_2=\frac{\rho B^4}{I}A_2^*(K),\ A_3=\frac{\rho B^4}{I}A_3^*(K),\ A_4=\frac{\rho B^3}{I}A_4^*(K) \tag{3b}$$

then Eq. (1) is rewritten as

$$\ddot{h}+2\xi_h\omega_h\dot{h}+\omega_h^2h=\omega H_1\dot{h}+\omega H_2\dot{\alpha}+\omega^2H_3\alpha+\omega^2H_4h \tag{4a}$$

$$\ddot{\alpha}+2\xi_\alpha\omega_\alpha\dot{\alpha}+\omega_\alpha^2\alpha=\omega A_1\dot{h}+\omega A_2\dot{\alpha}+\omega^2A_3\alpha+\omega^2A_4h \tag{4b}$$

Setting $\mathbf{x}(t)=[h(t)\ \alpha(t)]^T$, then Eq. (4) can be written in a matrix style as

$$\ddot{\mathbf{x}}+\overline{\mathbf{C}}\dot{\mathbf{x}}+\overline{\mathbf{K}}\mathbf{x}=\omega^2\mathbf{K}_{se}\mathbf{x}+\omega\mathbf{C}_{se}\dot{\mathbf{x}} \tag{5}$$

where

$$\overline{\mathbf{C}}=\begin{bmatrix}2\xi_h\omega_h & \\ & 2\xi_\alpha\omega_\alpha\end{bmatrix},\quad \overline{\mathbf{K}}=\begin{bmatrix}\omega_h^2 & \\ & \omega_\alpha^2\end{bmatrix}$$

$$\mathbf{C}_{se}=\begin{bmatrix}H_1 & H_2\\ A_1 & A_2\end{bmatrix},\quad \mathbf{K}_{se}=\begin{bmatrix}H_4 & H_3\\ A_4 & A_3\end{bmatrix}$$

Let $\mathbf{x}(t)=\psi e^{\lambda t}$, where $\psi$ is the complex mode response of the system including the structure and airflow; its corresponding complex frequency $\lambda=(-\xi+i)\omega$ (where $\xi$ and $\omega$ are the damping ratio and circular frequency of the complex mode, respectively, and $i^2=-1$), substituting this into Eq. (5) yields

$$(\lambda^2+\lambda\overline{\mathbf{C}}+\overline{\mathbf{K}})\psi e^{\lambda t}=(\omega^2\mathbf{K}_{se}+\omega\lambda\mathbf{C}_{se})\psi e^{\lambda t} \tag{6}$$

In order to solve the characteristics of complex modes of the system by linear eigensolver, for the exponential $e^{\lambda t}\neq 0$ the characteristic equation is given as

$$\left[\lambda^2\left(\mathbf{I}+\frac{1}{\lambda}\overline{\mathbf{C}}-\frac{\omega}{\lambda}\mathbf{C}_{se}-\frac{\omega^2}{\lambda^2}\mathbf{K}_{se}\right)+\overline{\mathbf{K}}\right]\boldsymbol{\psi}=0 \tag{7}$$

Denoting $\tilde{\mathbf{M}}=\mathbf{I}+1/\lambda\overline{\mathbf{C}}-\omega/\lambda\mathbf{C}_{se}-\omega^2/\lambda^2\mathbf{K}_{se}$, where $\mathbf{I}$ is a unit matrix. The analysis of complex modes of the system is converted into the following standard eigenvalue problem:

$$\overline{\mathbf{K}}\boldsymbol{\psi}=-\lambda^2\tilde{\mathbf{M}}\boldsymbol{\psi} \tag{8}$$

when the reduced frequency $K$ is known, four pairs of eigenvalues and eigenvectors of the characteristic equation can be solved by a simultaneous iteration method. The conjugated eigenvalues are $\lambda_1$, $\lambda_2$, $\lambda_1^*$, $\lambda_2^*$ and the corresponding eigenvectors are $\psi_1$, $\psi_2$,$\psi_1^*$, $\psi_2^*$, which are the complex modal parameters of the system.

The free vibration responses of the system can be written as

$$\mathbf{x}(t)=\sum_{r=1}^{2}(c_r\psi_r e^{\lambda_r t}+c_r^*\psi_r^* e^{\lambda_r^* t}) \tag{9}$$

where $c_r$, $c_r^*$ are constant and are determined by initial conditions. Since $\mathbf{x}(t)$ are real number functions, $c_r^*$ is conjugated to $c_r$.

Let $\lambda_r=\sigma_r+i\beta_r$, $\lambda_r^*=\sigma_r-i\beta_r$, then

$$\mathbf{x}(t)=\sum_{r=1}^{2}e^{\sigma_i t}[\mathbf{u}_r\cos(\beta_r t)+\mathbf{v}_r\sin(\beta_r t)] \tag{10}$$

where $\mathbf{u}_r=\mathrm{Re}(2c_r\psi_r)$, $\mathbf{v}_r=\mathrm{Im}(-2c_r\psi_r)$

The aforementioned discussion is the modal analysis of the system vibration, but the identification of the flutter derivatives in the following part is its inverse problem. The parameters of the two complex modes of the system can be extracted by a modified least-square method (Ding et al., 2001), which is briefly introduced in the following parts. When the complex modal parameters are known at one reduced frequency $K$, in order to determine the matrices of the self-excited forces the characteristic equations are written as

$$(\omega_i^2\mathbf{K}_{se}+\omega_i\lambda_i\mathbf{C}_{se})\psi_i=(\lambda_i^2+\lambda_i\overline{\mathbf{C}}+\overline{\mathbf{K}})\psi_i \tag{11}$$

thus

$$\begin{bmatrix}\mathbf{K}_{se} & \mathbf{C}_{se}\end{bmatrix}\begin{Bmatrix}\omega_i^2\psi_i\\ \omega_i\lambda_i\psi_i\end{Bmatrix}=(\lambda_i^2+\lambda_i\overline{\mathbf{C}}+\overline{\mathbf{K}})\psi_i \tag{12}$$

The above equations of four complex modes can be written as a matrix style, and multiplying the inverse matrix, then

$$\begin{bmatrix}\mathbf{K}_{se} & \mathbf{C}_{se}\end{bmatrix}=\begin{bmatrix}\mathbf{\Phi}(\lambda_i^2+\lambda_i\overline{\mathbf{C}}+\overline{\mathbf{K}}) & \mathbf{\Phi}^*(\lambda_i^{*2}+\lambda_i^*\overline{\mathbf{C}}+\overline{\mathbf{K}})\end{bmatrix}\begin{bmatrix}\mathbf{\Phi}\mathbf{W}^2 & \mathbf{\Phi}^*\mathbf{W}^2\\ \mathbf{\Phi}\mathbf{\Lambda}W & \mathbf{\Phi}^*\mathbf{\Lambda}^*\mathbf{W}\end{bmatrix}^{-1} \tag{13}$$