J. Wind Eng. Ind. Aerodyn. 99 (2011) 267–278

Contents lists available at ScienceDirect

Journal of Wind Engineering and Industrial Aerodynamics

journal homepage: www.elsevier.com/locate/jweia

Improved k–ε model and wall function formulation for the RANS simulation of ABL flows

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ARTICLE INFO

Available online 4 February 2011

Keywords:
ABL
Rough wall function
Turbulent kinetic energy
k-Epsilon turbulence model
RANS

ABSTRACT

The simulation of Atmospheric Boundary Layer (ABL) flows is usually performed using the commercial CFD codes with RANS turbulence modelling and standard sand–grain rough wall functions. Such approach generally results in the undesired decay of the velocity and turbulent profiles specified at the domain inlet, before they reach the section of interest within the computational domain. This behaviour is a direct consequence of the inconsistency between the fully developed ABL inlet profiles and the wall function formulation.

The present paper addresses the aforementioned issue and proposes a solution to it. A modified formulation of the Richards and Hoxey wall function for turbulence production is presented to avoid the well-documented over-prediction of the turbulent kinetic energy at the wall. Moreover, a modification of the standard k–ε turbulence model is proposed to allow specific arbitrary sets of fully developed profiles at the inlet section of the computational domain.

The methodology is implemented and tested in the commercial code FLUENT v6.3 by means of the User Defined Functions (UDF). Results are presented for two neutral boundary layers over flat terrain, at wind tunnel and full scale, and for the flow around a bluff-body immersed into a wind-tunnel ABL. The potential of the proposed methodology in ensuring the homogeneity of velocity and turbulence quantities throughout the computational domain is demonstrated.

1. Introduction

The limitations related to the Reynolds-averaged Navier–Stokes (RANS) simulation of the neutral atmospheric boundary layer (ABL) with the commercial CFD codes are well known and documented in the literature (Franke et al., 2007; Blocken et al., 2007a, b; Riddle et al., 2004; Hargreaves and Wright, 2007). The cause of such unsatisfactory behaviour is directly related to inconsistencies between the formulation of the law of the wall for rough surfaces and the inlet conditions for the ABL simulations. Remedial measures have been proposed in the literature (Blocken et al., 2007b); however, these are generally code-dependant and do not provide a general solution to the problem. In particular, the effect of roughness on turbulent quantities is not explicitly taken into account, causing an undesired non-homogeneity of the turbulent quantities throughout the computational domain.

As far as inlet profiles for the ABL simulations are concerned, fully developed profiles by Richards and Hoxey (1993) are usually used as inlet conditions. However, the assumption of constant kinetic energy, k, is not consistent with wind-tunnel measurements (Leitl, 1998; Xie et al., 2004; Yang et al., 2009), where a variation of k with height is generally observed. Following these considerations, Yang et al. (2009) proposed a new set of inlet conditions, where the k profile is a function of height; however, the consistency between this new set of inlet conditions and the k–ε model equations was not completely tackled. In a recent work, Gorlé et al. (2009) proposed formulations for the C_μ and σ_ε turbulence model constants to ensure stream-wise homogeneity when using the k profile proposed by Yang et al. (2009).

In case of full-scale ABL applications, semi-empirical correlations are provided to estimate the level of turbulent kinetic energy, based on the ABL friction velocity and height (Brost and Wyngaard, 1978). However, modifications need to be brought to the turbulence model, to ensure that the resulting set of fully developed profiles satisfies the transport equations.

The objective of the present paper is to develop an improved k–ε turbulence model for the numerical simulation of neutral ABL flows, with arbitrary sets of fully developed inlet conditions. This is

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0167-6105/$ - see front matter
doi:10.1016/j.jweia.2010.12.017

accomplished through the introduction of two source terms in the transport equations of k and ε, to ensure that the profiles identically satisfy the ABL model equations. Moreover, the overall consistency of the approach is ensured through a novel implementation of a general-purpose wall function for rough surfaces, based on the aerodynamic roughness, as indicated by Richards and Hoxey (1993).

2. Theory

The equations describing a 2-dimensional ABL with the standard k–ε model, under the hypothesis of (i) zero vertical velocity, (ii) constant pressure along vertical and stream-wise directions and (iii) constant shear stress, reduce to:

$$\mu_T \frac{\partial u}{\partial z} = \tau_w = \rho u^{*2} \tag{1}$$

$$\frac{\partial}{\partial z}\left(\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial z}\right) + G_k - \rho\varepsilon = 0 \tag{2}$$

$$\frac{\partial}{\partial z}\left(\frac{\mu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial z}\right) + C_{\varepsilon 1} G_k \frac{\varepsilon}{k} - C_{\varepsilon 2}\rho\frac{\varepsilon^2}{k} = 0 \tag{3}$$

where the dynamic viscosity has been neglected with respect to the turbulent viscosity ($\mu_t \gg \mu$). The production of turbulent kinetic energy, G_k, and the turbulent viscosity, μ_t, are given by

$$G_k = \mu_t\left(\frac{\partial u}{\partial z}\right)^2, \quad \mu_t = \rho C_\mu \frac{k^2}{\varepsilon} \tag{4}$$

and σ_k, σ_ε, $C_{\varepsilon 1}$, $C_{\varepsilon 2}$ and C_μ are constants of the k–ε turbulence model.

To correctly simulate a fully developed ABL, the velocity, turbulent kinetic energy and turbulent dissipation rate profiles specified at the inlet boundary should satisfy Eqs. (1)–(3). The consistency between fully developed inlet boundary conditions and turbulence model formulation is discussed in Section 2.1. In addition, it should be guaranteed that the boundary condition applied at the wall correctly represents the influence of the surface roughness. This topic is discussed in Section 2.2.

2.1. Improved k–ε turbulence model formulation

In the framework of ABL simulations, fully developed inlet profiles for velocity and turbulent quantities are generally prescribed at the inlet section of the computational domain. Mathematically, this implies that these profiles identically satisfy Eqs. (1)–(3).

Richards and Hoxey (1993) proposed profiles of mean velocity, turbulent kinetic energy and dissipation rate for neutral stratification conditions:

$$u = \frac{u^*}{\kappa}\ln\left(\frac{z+z_0}{z_0}\right) \tag{5}$$

$$k = \frac{u^{*2}}{\sqrt{C_\mu}} \tag{6}$$

$$\varepsilon = \frac{u^{*3}}{\kappa(z+z_0)} \tag{7}$$

It can be shown that Eqs. (5)–(7) are analytical solutions of the momentum and k–ε model equations if the turbulent dissipation number Prandtl number is given by

$$\sigma_\varepsilon = \frac{\kappa^2}{(C_{\varepsilon 2} - C_{\varepsilon 1})\sqrt{C_\mu}} \tag{8}$$

The condition expressed by Eq. (8) is obtained by substituting the imposed profiles of turbulent kinetic energy and dissipation rate in the transport equation for ε and solving for σ_ε. Alternatively, the constant value of σ_ε can be maintained and a source term added to the dissipation rate equation:

$$S_\varepsilon(z) = \frac{\rho u^{*4}}{(z+z_0)^2}\left(\frac{(C_2 - C_1)\sqrt{C_\mu}}{\kappa^2} - \frac{1}{\sigma_\varepsilon}\right) \tag{9}$$

A similar approach was adopted by Pontiggia et al. (2009) who retained, however, the molecular viscosity in their derivation. Different from Eq. (8), the introduction of a source term in the transport equation for ε does not require the calculation of σ_ε for each computation as S_ε self-adapts to the characteristics of the ABL under investigation.

The fully developed profiles provided by Richards and Hoxey (1993) are mathematically consistent, i.e. they are a solution of the mathematical models describing a homogeneous ABL. However, the choice of a constant k profile (Eq. (6)) contradicts the experimental evidence showing a trend in decreasing k with height (Leitl, 1998; Xie et al., 2004; Yang et al., 2009). Recently, Yang et al. (2009) analytically derived an inlet condition for turbulent kinetic energy as a function of the height from the ground:

$$k(z) = \sqrt{C_1 \ln(z+z_0) + C_2} \tag{10}$$

where C_1 and C_2 are constants that can be determined by fitting the measured profile of k. By assuming equilibrium between turbulent production and dissipation, the turbulent dissipation rate profile can be expressed as

$$\varepsilon(z) = \sqrt{C_\mu} k \frac{du}{dz} \tag{11}$$

As far as the consistency between fully developed inlet conditions and turbulence model is concerned, Yang et al. (2009) argued that the profile expressed by Eq. (10) identically satisfies the transport equation for k. However, they did not take into account the implications of a non-constant k profile on the momentum and turbulent dissipation rate transport equation. In particular, a general condition on the turbulence model parameter C_μ can be deduced substituting Eq. (11) into Eq. (1) and employing the definition of turbulent viscosity (Eq. (4)):

$$C_\mu(z) = \frac{u^{*4}}{k(z)^2} \tag{12}$$

Eq. (12) is simply the relation proposed by Richards and Hoxey (1993) inverted, to ensure consistency between the turbulence model, i.e. C_μ, and the k profile throughout the ABL domain. From the point of view of the physical interpretation, the non-uniform k profile and the definition of C_μ can be related to the large-scale turbulence present in the ABL flows, which can vary significantly with height. Bottema (1997) indicated the relevance of large-scale turbulence to several RANS models, pointing out the necessity for case- and location-dependant model constants. The present paper follows this approach proving a model defining optimal local values of C_μ, according to Eq. (12). It is also noted that the assumption of equilibrium between production and dissipation, which results in the proposed relation for C_μ, implies that the gradient of turbulent kinetic energy does not introduce vertical diffusion of k. This assumption was justified by estimating the magnitude of the diffusion term based on the experimental data, which resulted negligible compared to the production of turbulence kinetic energy.

In addition to specifying C_μ according to Eq. (12), a source term is added to the transport equation of turbulent kinetic energy, to ensure equilibrium between production and dissipation. This is due to the fact that for a non-uniform C_μ, the analytic profile derived by Yang et al. (2009) is no longer a solution of the k transport equation and the following extra term appears when