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Numerical analysis of added mass for open flat membrane vibrating in still air using the boundary element method

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ABSTRACT

Added mass has significant effect on the vibration of membrane structures, which cannot be ignored during vibration analysis. Actually, the added mass of typical objects, such as cylinders and spheres, moving in fluid with acceleration have been widely investigated. However, research on the added mass of flexible structures is still limited. Although several numerical methods had been developed to investigate the added mass of flexible structures, the efficiency of the numerical methods is not verified by experiment tests. In this study, a framework to numerically analysis the added mass of open flat membranes has been established by using the Boundary Element Method (BEM). In order to evaluate hypersingular integral, the Stokes formula converting the surface integral to the curvilinear integral is used. Two added mass models are discussed, one only considering the effect of the membrane geometric shape, and another considering the effect of the geometric shape and the mode shape of membranes. Based on comparative analysis between numerical results and experimental results, it is shown that the added mass only considering the effect of geometric shape can agree well with the test results in low-order modes, however, the error will be increased as the order of vibration modes increasing. The added mass considering the effect of the geometric shape and the mode shape can have much better conformity with the test results both in low-order modes and high-order modes. The Modal Assurance Criterion (MAC) is used to compare the mode shapes of membranes vibrating in vacuum and in air. MAC values indicate that for uniform mass distribution of the membrane, there is little difference between the mode shapes of the membrane vibrating in vacuum and in air, while for nonuniform mass distribution of the membrane, the difference between the mode shapes of the membrane vibrating in vacuum and in air is little in low modes and is large in high modes.

1. Introduction

In fluid mechanics, the added mass or virtual mass, is the added inertia to the system, since the increase or decrease in the body acceleration should cause the fluid to move around the body in such a way that the object can move through it, and the body and the fluid cannot simultaneously occupy this physical space. For light weight structures, such as membrane structures, when they vibrate in a certain kind of fluids, a part of the surrounding fluid will be invoked and will vibrate together with structures. Hence, the added mass should have a significant influence on the vibration of membrane structures.

Actually, the added mass of typical objects, such as cylinders and spheres, moving in fluid with acceleration had been widely investigated. Some determinations of the fluid loading and the added mass for a supported plate are known from the slender wing theory (Jones, 1946), the traveling wave solution (Miles, 1956; Dugundji et al., 1963), two-dimensional linear aerodynamic theory (Kornecki et al., 1976), or three-dimensional linear aerodynamic theory (Lucey and Carpenter, 1993). Yadykin et al. (2003) reviewed the fluid loading formulations and applied the thin airfoil theory to numerical study of the fundamental properties of the added mass of a flexible plate oscillating in fluid. However, the research on the added mass of membrane structures is still limited. Up to now, several numerical simulation methods had been developed to investigate the added mass of membrane structures. Irwin and Wardlaw (1979) presented an empirical equation for estimating the added mass for the membrane roof of Montreal Stadium. With the framework of the thin airfoil theory, Minami (1998) had investigated a membrane with its ends fixed in an incompressible fluid, and it was proposed that the added mass is equivalent to the air uniformly distributed on the membrane

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with an estimated height of 68% the length of the membrane. Sygulski (1993) presented a method for solving the free vibration and the linear forced harmonic vibration problems for pneumatic structures interacting with air. The Finite Element Method (FEM) for the structure and the Boundary Element Method (BEM) for the air were used. Sygulski (1994) presented a method for solving the free vibration and the linear forced harmonic vibration problems for open membrane structures interacting with air, and the method describing the aerodynamic pressure was based on the boundary integral equation, which was solved by the BEM. Sygulski (1997) analyzed the problem of interaction between a pneumatic structure and the surrounding air by using the BEM and FEM. However, no test result had been applied to verify the results by the numerical simulation methods mentioned above. Sewall et al (1983) undertook an experimental investigation of membrane vibrations. Tests were preformed both in air and in vacuum for various membrane pretensions. Sewall et al. (1983) also proposed a distribution model of the added mass of the membrane. Li et al (2011) tested the vibration of a circular flat membrane in still air with varying air pressures, and a simplified added mass model was proposed based on the vibration mode shapes of the flat membranes, i.e., the added mass above each vibration region is equal to the uniformly distributed air with height of 0.65l, in which l is the diameter of the inscribed circle of the region. The added mass coefficient, 0.65, was derived from the fitting analysis of the circular membrane results, and was also proved by the existing test data of a three-sided membrane by Sewall et al (1983). However, this simplified added mass model was lack of theoretical analysis.

In this study, a framework to numerically analyze the added mass of open flat membranes has been established by using the BEM. The velocity potential of the still air satisfies the Laplace equation, and the boundary conditions on the surface are of the Neumann type. The aerodynamic pressure is described by the boundary integral equation, and solved by the BEM. In order to evaluate hypersingular integral, the Stokes formula converting the surface integral to the curvilinear integral is used. Two added mass models are discussed, one only considering the effect of the membrane geometric shape, and another considering the effect of both the geometric shape and mode shape of membranes. The numerical results of two added mass models are compared with the data of the existing tests on circular, square and three-sided membranes. The Modal Assurance Criterion (MAC) is a statistical indicator that is most sensitive to large differences and relatively insensitive to small differences in the mode shapes (Randall, 2003; Pastor et al., 2012). It is bounded between 0 and 1, with 1 indicating fully consistent mode shapes. A value near 0 indicates that the modes are not consistent. In this paper, the MAC is used to indicate the difference of mode shapes between membranes vibrating in vacuum and in air.

2. The added mass of open flat membranes

2.1. Numerical analysis of still air induced by open flat membrane

A light open membrane structure of any shape in still air is considered. The membrane during vibration will induce the motion of surrounding air, and the air becomes a source of the additional inertia forces, as the same as the contribution of the structural mass. It is assumed that the air is incompressible and inviscid, and the velocity potential of the air satisfies the Laplace equation, i.e.,

$$\frac{\partial^2 \boldsymbol{\varphi}}{\partial x^2}+\frac{\partial^2 \boldsymbol{\varphi}}{\partial y^2}+\frac{\partial^2 \boldsymbol{\varphi}}{\partial z^2}=0 \tag{1}$$

where $\boldsymbol{\varphi}$ is the velocity potential of the air.

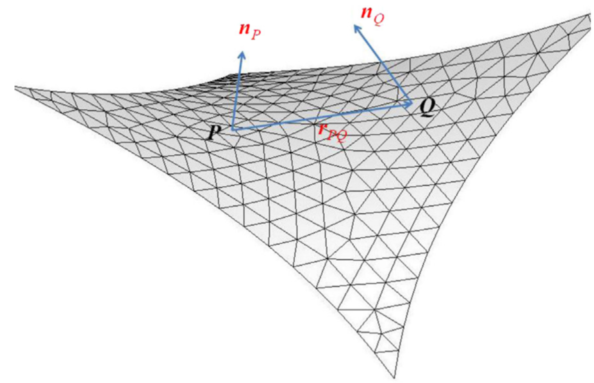

Fig. 1. A light open membrane structure.

The solution of this equitation in integral form is

$$4\pi\frac{\partial \boldsymbol{\varphi}_P}{\partial \boldsymbol{n}_P}=\iint_{\Gamma_i}\boldsymbol{\varphi}_Q\frac{\partial^2}{\partial \boldsymbol{n}_P\partial \boldsymbol{n}_Q}\left(\frac{1}{\boldsymbol{r}_{PQ}}\right)d\Gamma \tag{2}$$

where, $\boldsymbol{\varphi}_Q$ is the velocity potential of the air at point Q on the surface, $\boldsymbol{\varphi}_P$ is the velocity potential in any point of the space, $\boldsymbol{r}_{PQ}$ is the distance between any point P and a point Q on the surface (as shown in Fig. 1), and $\partial\boldsymbol{\varphi}_P/\partial\mathbf{n}_P$ is the air velocity normal to the surface at the point P.

The boundary condition on the surface S is of Neumann's type and it is a coupling condition between the structure and the air. The formulations of the aerodynamic pressure and acceleration of the air are

$$\boldsymbol{p}_n=-\rho\frac{\partial \boldsymbol{\varphi}}{\partial t},\quad \boldsymbol{a}_n=\frac{\partial^2 \boldsymbol{\varphi}}{\partial n \partial t} \tag{3}$$

where ρ is the air density.

Differentiating Eq. (2) with respect to time and using Eq. (3) yields

$$-4\pi\boldsymbol{a}_{nP}=\iint_{\Gamma_i}\mathbf{p}_{nQ}\frac{\partial^2}{\partial \boldsymbol{n}_P\partial \boldsymbol{n}_Q}\left(\frac{1}{\boldsymbol{r}_{PQ}}\right)d\Gamma \tag{4}$$

where $\boldsymbol{p}_{nQ}$ is the resultant aerodynamic pressure acting at the point Q.

The BEM is used to numerically solve the boundary integral equation, Eq. (4). The surface of the membrane structure is discretized using the triangular elements. The boundary element discretization of Eq. (4) results in the following equation:

$$-4\pi\boldsymbol{a}_n=\boldsymbol{A}\boldsymbol{p}_n \tag{5}$$

where, the matrix $\boldsymbol{A}$, a $N\times N$ complex matrix (N is the number of triangular elements) is

$$\boldsymbol{A}=\iint_{\Gamma_i}\frac{\partial^2}{\partial \boldsymbol{n}_P\partial \boldsymbol{n}_Q}\left(\frac{1}{\boldsymbol{r}_{PQ}}\right)d\Gamma \tag{6}$$

The kernel of the integral has a strong singularity of the r^{-3} order, when the point Q approaches the point P ($\boldsymbol{r}_{PQ}\to 0$). For this case the integral cannot be directly determined. It can construct a set of related functions and use the Stokes formula to convert to the curvilinear integral on the edge of the surface. In this way, both the computational efficiency and accuracy are improved greatly.

The differentiation in Eq. (6) can be perform in the following form

$$\frac{\partial^2}{\partial \boldsymbol{n}_P\partial \boldsymbol{n}_Q}\left(\frac{1}{\boldsymbol{r}_{PQ}}\right)=\frac{-3z_P(x_Q-x_P)}{r_{PQ}^5}n_x+\frac{-3z_P(y_Q-y_P)}{r_{PQ}^5}n_y+\frac{-r_{PQ}^2+3z_P^2}{r_{PQ}^5}n_z \tag{7}$$

As shown in Fig. 2, the local coordinate system of the element P is defined by (ξ, η, ζ). The direction of the unit axis vectors ζ is the