Mining Science and Technology (China) 21 (2011) 755–759

Contents lists available at SciVerse ScienceDirect

Mining Science and Technology (China)

journal homepage: www.elsevier.com/locate/mstc

Study on the chaotic behavior of mining rock seepage system

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ARTICLE INFO

Article history:
Received 13 February 2011
Received in revised form 10 April 2011
Accepted 9 May 2011
Available online 20 January 2012

Keywords:
Non-Darcy flow
Chaos
Power spectrum
Correlation dimension
Lyapunov indices

ABSTRACT

One dimensional non-steady, non-Darcy flow of water in a rock stratum was reduced into a system described by six ordinary differential equations involving five controlling parameters. Through response computations and time series analysis, chaotic behavior in the reduced system was discussed in details. Firstly, the dynamical response of the reduced system under a set of parameters was calculated, and the power spectrum of the attractor was obtained through fast Lagrangian transformation; then the phase space was reconstructed by fixing embedding dimension to be 6 and delay time to range from 1 to 20, and the correlation dimension of the attractor was calculated based on the curves under the coordinates of logarithm of correlation integral vs. logarithm of covering radius; and lastly, the Lyapunov indices of the attractor were calculated by using Gram-Schmit's orthogonalization method. The results show that the power spectrum of the attractor is continuous; the correlation dimension of the attractor is equal to 2.36; among the Lyapunov indices, LE1, LE2, LE3 are positive, LE5, LE6 are negative, and LE4 fluctuates near zero. All the analysis indicates that there may exist chaos in the system of non-steady, non-Darcy flow.

1. Introduction

Mining rock seepage mechanics, which is the basic theory of studying happing mechanism of water and gas outbursts in coal mine, is a branch of mining rock mechanics [1,2]. Since it starts later, the mining rock permeation fluid mechanics has not been formed well and the research results are not very rich, which even does not combine the engineering practice closely. Therefore, it is an urgent task to develop the mining rock permeation fluid mechanics.

The seepage of water or gas in mining rock/coal seams has two characteristics, one is unsteady-state and the other is nonlinearity. Therefore, nonlinear dynamics should be the basic tools for studying the mining rock permeation fluid mechanics. The structure stability of nonlinear and unsteady-state seepage system is the main studying contents for mining rock permeation fluid mechanics. The existing research results show that mining rock seepage system has possibility to bifurcate (namely the topology structure change of phase orbit) [3–5]. Though the structure stability of mining rock seepage system can express the mechanism of water and gas outbursts, the point is not been got response and attention in academia [6,7].

The nonlinear and unsteady-state seepage system also can appear chaos, which means that it is impossible for long-term forecast for water and gas outbursts. This paper uses the method of time series analysis and discusses the existence of chaos in seepage system. That is, through the power spectrum, correlative dimension and Lyapunov index to decide the appearance of chaos when the controlling parameters meet certain conditions in seepage system.

Mining rock refers to the part of rock mass influenced by mining stress with mining ore body. During coal mining, in general, the over-layering strata will break and cause the surface movement. Therefore, the mining rock often consists of various massive rock after damage, of which the notable mechanical behavior feature is break and rigid motion. Mining rock is an interdiscipline subject combining the solid mechanics with mining engineering. Since the studying range of mining rock mechanics is the carrying capacity after damage, it breaks through the frame of solid mechanics and provides sharp challenge for solid mechanics.

In order to study the motion and break laws of mining surrounding rock, the mining technical workers proposed pressure arch hypothesis, cantilever beam hypothesis, pre-splitting fissure hypothesis, hinged rock hypothesis, voussoir beam hypothesis, transferring rock beam model and so on [2]. Especially, the theory of key strata proposed by Qian et al. provides the frame for mining rock mechanics. The theory of key strata considers that, the over-layering strata will form carrying structure after mining coal seam and this structure can be seen as multi-layer stacked beam or board because of the layering characteristics of over-layering strata. Since the sedimentary processes of various rock layers are

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1674-5264/$ - see front matter © 2012 Published by Elsevier B.V. on behalf of China University of Mining & Technology.
doi:10.1016/j.mstc.2011.12.001

different, their mineral composition and mechanical properties are different, and the effects are also different. The main cause of a series of mine pressure behavior laws including deformation break and separated layer of over-layering strata is the key strata in hard stratum. The key strata are elastic foundation beam before break, but become voussoir beam structure after break. The key strata theory for control of strata provides theoretical basis for further study of distribution laws of strata motion and mining fracture. The key strata theory has been widely used in mining engineering field, for example, the application of key strata composition effect to mine pressure, the application of key strata theory to relief-pressure gas extraction and the prevention of water inrush in floor [8–11]. In the 20th century, the key strata theory has developed further and its landmark achievement is the compound key strata theory [12–14]. In addition, the application of combing key strata theory and variable boundaries systemic dynamics to the description and numerical simulation of over-layering strata breaking process also got some research results [15–17].

Due to the need of protecting groundwater resources in arid and semiarid area of western, many academics put forward the concept of green mining for coal resources, which provides a new dynamic for the development of key strata theory. In this context, the concepts of water-resisting key strata and seepage key strata have been proposed [18]. In recent years, the study on nonlinear behavior of seepage system (such as bifurcation, chaos, mutation) also has made many progresses. Wang et al. proposed a cusp catastrophe model of water inrush in floor and analyzed the mechanism of water inrush and instability failure [19]. Miao et al. discussed the bifurcation behavior of non-Darcy seepage system after peak by using spectrum truncation and the first approximate theory of Lyapunov and established the systematic bifurcation conditions [3]. Using separation of variables, Sun et al. studied the instability conditions of water seepage in layered rock mass and analyze the mechanism of water inrush from the point of structural stability [6]. Kong et al. defined the coefficients of water inrush and showed the conditions of water inrush by water inrush coefficients [20,21]. There are complex couplings between seepage and deformation of surrounding rock, including the influences of surrounding rock deformation on seepage characteristics, pressure distribution, seepage velocity and the influences of pore pressure on surrounding rock deformation. A number of mechanical problems in mining engineering need to consider the effect of flow-solid coupling to meet the requirements of engineering practice.

For general geotechnical engineering, seepage is extreme slow flow, which is often studied as a liner and steady-state process. However, in the mining engineering, the seepage in surrounding rock often appears instability and causes disasters of water inrush, which brings great damage to safety production for coal mine. The seepage of mining rock has the features of instability, nonlinear and time-varying parameters [1]. Therefore, it is of great significance in theory and engineering to study the seepage laws of surrounding rock, reveal the conditions of seepage instability, forecast the probability of water inrush, and improve the safety production and the economic benefits of coal mine by using the point of nonlinear dynamics.

In seepage system of mining rock, the controlling parameters include permeability, the coefficient β of non-Darcy flow, the dynamic viscosity of water and rock thickness. When these parameters meet certain conditions, the system appears the "chaos" and the long-term forecasts of dynamic behavior are impossible. It can avoid the occurrence of chaos by control of stratum (such as support and grout) to change the controlling parameters of system. Therefore, studying the chaos behavior of seepage system has vital significance for prevent the occurrences of disasters, such as water inrush.

2. Dynamic model of seepage system in mining surrounding rock

Discuss one-dimensional seepage of water in single strata, assuming that the penetration properties of strata distribute uniformly along the high direction and do not change with time, and then the control equation of seepage can be simplified as [1]

$$\begin{cases} \phi_0 c_t \frac{\partial p}{\partial t} + \frac{\partial V}{\partial x} = 0 \\ \rho c_a \frac{\partial V}{\partial t} = -\frac{\partial p}{\partial x} - \frac{\mu}{k} V - \beta\rho V^2 \end{cases} \tag{1}$$

where x shows the coordinates of flow direction, μ is the dynamic viscosity of water; k, β, c_a show permeability of strata, coefficient β of non-Darcy flow and acceleration coefficient respectively; $c_t = c_f + c_\phi$ is integrated compression coefficient, c_f is the compression coefficient of water and c_φ is the pore compression coefficient of strata; φ_0 and ρ are the rock porosity and quality density of water in the condition of $p = p_0$.

In Ref. [1], by using Garlerkin method, the system (1) reduces order to system of differential equations with six variables and five controlling parameters and the model after reducing order is [1]

$$\begin{cases} \dot{y}_1 = \pi a_0 y_4 \\ \dot{y}_2 = 2\pi a_0 y_5 \\ \dot{y}_3 = 3\pi a_0 y_6 \\ \dot{y}_4 = -\pi a_1 y_1 - a_2 y_4 + \frac{8}{3} a_4 \tilde{W}_s y_5 + \frac{2}{3} a_4 y_4^2 + \frac{8}{15} a_4 y_5^2 + \frac{18}{35} a_4 y_6^2 \\ \qquad + a_3 y_4 y_5 + \frac{12}{5} a_4 y_4 y_6 + a_3 y_5 y_6 \\ \dot{y}_5 = -2\pi a_1 y_2 - \frac{2}{3} a_4 \tilde{W}_s y_4 - a_2 y_5 + \frac{18}{5} a_4 \tilde{W}_s y_6 \\ \qquad + \frac{a_3}{2} y_4^2 + \frac{22}{15} a_4 y_4 y_5 + a_3 y_4 y_6 + \frac{6}{7} a_4 y_5 y_6 \\ \dot{y}_6 = -3\pi a_1 y_3 - \frac{8}{5} a_4 \tilde{W}_s y_5 - a_2 y_6 \\ \qquad - \frac{2}{5} a_4 y_4^2 + \frac{8}{7} a_4 y_5^2 + \frac{2}{3} a_4 y_6^2 + a_3 y_4 y_5 \end{cases} \tag{2}$$

where $y_i (i = 1, 2, \cdots, 6)$ is the state variables; a_0, a_1, a_2, a_3, a_4 express the controlling parameters and

$$a_0 = \frac{1}{p_0 \phi_0 c_t}, \quad a_1 = \frac{p_0 \rho_0 \beta^2 k^2}{c_a \mu^2}, \quad a_2 = \frac{\beta l}{c_a}\sqrt{1 - \frac{4\beta k^2 \rho_0 p_0}{\mu^2 l}}, \quad a_3 = \frac{\beta l}{c_a}, \quad a_4 = \frac{c_f}{\phi_0 c_t} \tag{3}$$

where l is the rock thickness; p_0 shows the pressure in bottom of strata (the pressure in upper strata is zero); $\tilde{W}_s$ is dimensionless momentum density of system steady motion

$$\tilde{W}_s = \frac{1}{2}\left(1 \pm \frac{a_2}{a_3}\right).$$

The chaos of research system (2) needs to discuss the topology structure and properties of system reaction, structure stability (bifurcation), power spectrum, correlative dimension and Lyaumnov index in five dimensions parameter space (a_0, a_1, a_2, a_3, a_4) or (k, β, c_a, l, p_0). Under the condition of $a_4 = 0$ (namely ignoring the compressibility of water), the eigenvalues of linear system, which is corresponding to system (2), are as follows [1]:

$$\begin{cases} \lambda_{1,2} = \frac{-a_2 \pm \sqrt{a_2^2 - 4\pi^2 a_0 a_1}}{2} \\ \lambda_{3,4} = \frac{-a_2 \pm \sqrt{a_2^2 - 16\pi^2 a_0 a_1}}{2} \\ \lambda_{5,6} = \frac{-a_2 \pm \sqrt{a_2^2 - 36\pi^2 a_0 a_1}}{2} \end{cases}$$

Due to $a_0 > 0$, $a_1 > 0$, the signs of real part of eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_6$ are determined by a_2, so $a_2 = 0$ is the condition of occurring bifurcation in system (2) [1]. There is no state on the