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Reliable reconstruction strategy with higher grid resolution for limited data tomography

M. Goswami [a,*], S. Shakya [c], A. Saxena [c], P. Munshi [b,c]

[a] University of California Davis, Davis, CA 95616, USA
[b] Nuclear Engineering & Technology Programme, Indian Institute of Technology, Kanpur, India
[c] Department of Mechanical Engineering, Indian Institute of Technology, Kanpur, India

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ABSTRACT

A computerized tomography (CT) procedure is presented for reliable image recovery from limited data with the aim to improve depiction of the inner profile. Proposed strategy focuses on attaining (a) accuracy and reliability of the reconstructed properties like the shapes, sizes and densities, and (b) stability in presence of noise by optimally choosing various reconstruction parameters pertaining to smearing and grid size. Sensitivity analysis is performed with different grid types and their spatial resolutions. Generalized spatial smearing is proposed to alleviate artifacts due to the presence of non-participating pixels. Best results are obtained when an optimal smearing window size (w.r.t. the l_2 norm) is employed. The proposed procedure is tested successfully for limited view data from both, the cyber domain and real world. It is observed that (i) explicit smearing is essential in limited data cases and (ii) optimal smearing parameters exist along with an interval of confidence within which reconstruction results can deemed to be reliable. We recommend using explicit optimal smearing for improved and reliable recovery from limited data. The contribution herein can be employed in the development of miniature CT setups for application where collection of full data is not possible.

1. Introduction

Computerized tomography (CT) is a non-invasive imaging technique employed to recover the internal details of an object. Amongst the inverse Fourier transform and iterative methods [1,2], the latter facilitate more acceptable results with datasets having relatively less number of rays/detectors or projections. These cases are often categorized into limited detector tomography (LDT) or limited view tomography (LVT) [3,4]. Practical circumstances, e.g., size of an object or tomographic setup impose limits on the number of sensors/detectors, where both cases (LVT and LDT) may occur. Such cases are termed here limited detector and view tomography (LDVT). Small ratio of detector aperture to diameter of the object does not allow using large number of detectors at times. Limited view tomography cases: (a) $<360^\circ$ rotation and (b) less number of rotations with full 360° view, have been discussed before in literature [1,5]. Combining these cases may present situations when projection data can be highly sparse [3–7]. Tomography modalities using CCD sensors such as X-ray and optical tomography are free from these problems but cannot be employed in all applications. Gamma ray, ultrasound, and electrical impedance tomography techniques, as alternative modalities, are under development [6].

A uniform mesh, e.g. square pixel grid (SPG) follows conventional discretization criteria. Nodes are located uniformly and their population is obtained by a criterion that ensures consistency in the system of equations. For LDVT cases, these norms provide diffused inner distribution in reconstructed image [3]. It thus behooves to improve a fixed meshing technique with better discretization parameters, e.g., higher number of pixels/nodes and their locations, and/or the interpolation method to retrieve properties at non-nodal locations [8]. Square pixel grid (SPG) and grids based on structured/unstructured Delaunay triangulation (DTS/DTU) are widely used in the imaging world [3,9]. Discretization parameters for the projection intersection (PI) grid [3] (briefly discussed later) are user independent.

1.1. Motivation

The effort herein is to examine sensitivity of various grids (SPG, DTS, DTU and PI)[1] for *comparatively higher grid ratio (GR) that goes beyond the conventional criteria* by combining the best option with a

* Corresponding author.
E-mail addresses: mayan@iitk.ac.in, mggm1982@gmail.com (M. Goswami).

[1] SPG: Square pixels grid, DTS: Delaunay Triangulation (structured), DTU: Delaunay Triangulation (unstructured), PI: projection interpolation grid.

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robust solution technique. Effect of the grid resolution on solution techniques is studied via simulation and verified on real data. Conventionally, smearing is performed by (a) shifting the detector between projections during the scan by certain (usually half) pixel width or/and (b) averaging the recovered variable under a certain size of mask. This latter case is discussed here. Quantification of user independent optimal smearing parameters against increment in grid resolution is explored. This step is convoluted with variables during the *reconstruction phase*. Any improvement, in reliability and accuracy in reconstruction, is expected to add towards development of a compact CT scanner for limited data cases where full data cannot be collected. A compact CT scanner, designed for constraints such as (a) detector size, (b) restricted budget, and (c) installation/application at remote location has advantages of (a) easy handling, (b) operation at comparatively low power and (c) flexibility to transport, etc. There is substantial demand in laboratory as well as industrial situations for such compact size CT scanners. Immediate application would be real time estimation of void fraction inside power plants having dense distribution of flow channels, slurry profile detection at oil industry and deep earth explorations (fracking) [6]. Study can be extended further for small size medical CT scanners, once the compact CT designs are improved and standardized. Secondary application may involve man power training in academic environment.

1.2. Organization

The maximum entropy method (MaxenT) is chosen for this *mix-and-match* exercise with these grids. Optimization is achieved using a derivative of quasi-Newton scheme [3,13]. Iterative methods such as Multiplicative algebraic reconstruction techniques (MART) require computational experience of handling the tuning parameters (for example initial guess, stopping criterion and relaxation factor) [1,8,14,15]. This method (MaxenT) is less sensitive for such algorithm parameters [3]. The following section discusses the definitions of solution techniques and interpolation aspects associated with the grids. A smearing operator, similar to the Gabor 2D filter function, is convolved in the discretization space [10–12] and discussed in Section 3. Optimal size of the smearing circular mask is quantified via an empirical strategy w.r.t. to minimum error as the grid resolution increases. The outcome of this analyses over synthetic data (Section 4) is validated on real world data in Section 5.

2. Algorithm

Framework of an iterative tomography algorithm can be divided into (a) discretization step and (b) solution step. Employed meshing and solution techniques are briefly discussed in this section.

2.1. Discretization

Formulations for the weight coefficients for SPG, DTS/DTU and PI grids appear elsewhere [1,3,9]. Significant number of inactive pixels (unmapped pixels) surface with increase in the number of nodes in the reconstruction grids [3]. Shape functions (employed in DTS/DTU) provide more involved interpolating relationship between the image and data spaces. They help to diminish the occurrence of inactive pixels with comparatively higher values of GR. However, the effect of inactive pixels cannot be alleviated entirely, even when interpolating shape functions are used. Number of inactive pixels increases significantly with further refinement in grid size. PI grid is preferred in later reconstruction exercises as inactive pixels do not appear and also, the grid resolution is fixed and dependent solely on the number of projection rays and views used to extract the LVDT sparse data.

2.2. Reconstruction methods

Entropy maximization (MaxenT) is combined with the SPG, STU, STS and PI grids for increasing number of nodes in case of the former three. MaxenT is modified further by convolving a smearing operator in-between the iterations (Section 4.1). A circular smearing mask is used whose radius R is chosen via empirical exercises. The modified approach is termed as sMaxenT. Optimality of smearing parameters is also addressed in Section 4.1. Implementation details are discussed elsewhere [3].

3. Smearing and improvements in the reconstruction algorithm

Conventionally, a filter window of 2×2 or 3×3 pixels is used in case of SPG. This grid is made of square elements (termed as pixels) with constant width. Grids with triangulated elements, however, do not have a repetitive pattern. Consequently, a square smearing window is not useful. A generalized function, similar to Nadaraya–Watson estimator (Eq. 1) is used herein [12,17–19]. The smearing parameter γ and radius R help to generate numerous smearing options. Sensitivity analysis is performed for (a) values of the smearing radius R between $10-40\%$ of the cross-section diameter and (b) values of γ between 0.1–2.6. This comprehensive approach is fairly liberal and captures a wide range of spatial filters. An empirical unbiased strategy is developed (Section 4.1) to choose an optimal filter w.r.t. least *RMS* error in reconstruction. It is helpful to estimate the best possible mask size for a given grid resolution. The effect of smearing on all secondary densities f_j over primary ones f_i reduces with distance d_{ij} between the ith and jth nodes (Fig. 1). The smeared density $\hat{f}_i$ corresponding to f_i is determined using the following operator:

$$\hat{f}_i=\frac{\sum_{j\ s.t.\ d_{ij}\le R} f_j\left(R-d_{ij}\right)^{\gamma}}{\sum_{j\ s.t.\ d_{ij}\le R}\left(R-d_{ij}\right)^{\gamma}} \quad (1)$$

Empirically optimized values (corresponding to minimum l_2 norm) of smearing radius R and exponent γ are expected to yield good results in reconstruction. The modified iterative algorithm is illustrated in a flow diagram in Fig. 2.

4. Simulation

Definitions for the weight coefficient differ with various interpolation techniques. To generate a phantom, average of the synthetic projection data by all four grids (DTS, DTU, SPG and PI) is taken to eliminate bias. It is observed that maximum and minimum density levels from these grids are approximately the same. Synthetic data must have sufficient information if fineness in the process of recovery is to be examined. One cannot expect good reconstruction results with a 40×40 pixels grid, if synthetic data itself is generated by, say a 9×9 pixel grid. This becomes an important aspect for limited data cases. We find, for this setup geometry, that a phantom of 60×60 pixels contains sufficient information as the *RMSE* in reconstruction for a given grid size saturates beyond this level. Reconstruction error for 9×9 pixel mesh is calculated while increasing the discretization level of the synthetic line integral. The circular cyber image Phantom1 (Fig. 3a) has four features (two different size off-centered circular regions, one rectangular and one square shaped high density impregnations) within which the linear attenuation/function value is 1. The surroundings have the value 0. Another synthetic image, Phantom2 (simulation in Fig. 3b) has binary