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Transient response of a driver-pickup coil probe in transient eddy current testing

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ABSTRACT

Novel solutions that correctly incorporate all electromagnetic interactions arising in inductively coupled circuits are presented for the case of a coaxial driver and pickup coil probe encircling a long ferromagnetic conducting rod. The differential circuit equations are formulated in terms of the rod's impulse response using convolution theory, and solved by Fourier transform. The solutions presented here are the first to account for feedback between a ferromagnetic conductor and the driver and pickup coils, providing correct voltage response of the coils. Experimental results, obtained for the case of square wave excitation, are in excellent agreement with the analytical equations.

1. Introduction

Driver-pickup eddy current non-destructive testing is a well-established method for the inspection of metallic objects [1]. It is relatively inexpensive, fast and reliable in comparison with other inspection technologies. These qualities make eddy current a desirable inspection method in industry [2,3], where it is employed to monitor the structural health of industrial assets. To this end, considerable efforts have been made to develop mathematical models that enable the interpretation of inspection data. The common approach is to formulate time-harmonic solutions, which describe the electromagnetic fields in a system, in order to calculate the change in a coil's impedance as it interacts with a conducting structure [4–8]. In a less developed approach, transient eddy current models consider the voltage induced in a pickup circuit [9–14] given a prescribed current that has been applied to a driver coil. Under voltage control, changing material characteristics and inspection geometries will distort the resultant current signal through feedback effects. In order to circumvent the feedback challenge, the common approach has been to employ current control systems. In such systems, however, the level of signal distortion from feedback effects is largely dependent on the quality of the current generator. Consequently, a persistent challenge for the development of transient driver-pickup models, particularly under voltage control, has been a lack of experimental agreement, particularly in cases where ferromagnetic materials, such as steel, exhibit stronger inductive coupling effects. In transient eddy current experiments, for instance, agreement with experimental data is limited to later times when feedback effects become less prominent [11]. In other cases, authors often assume non-magnetic samples for experimental validation [1–4], constrained sample geometries [11], and resort to fitted parameters [3] or convenient smoothing functions [9]. In some instances [9–13], analytical models have been presented with limited or no experimental support. In other cases, solutions are most frequently formulated for time-harmonic excitations for applications in conventional eddy current [15], and less often for general excitations, which include square waveforms for applications in pulsed eddy current.

Recently, a novel analytical approach, whereby electromagnetic field solutions are incorporated directly into Kirchhoff's circuit equations, and solved in terms of an applied voltage (instead of current), was developed [16]. This approach accounts for all electromagnetic interactions arising in inductively coupled systems and, thus, addresses the feedback problem. The theory was applied to the simple case of a driver coil encircling a ferromagnetic conducting rod [17]. The coil's calculated response to a step excitation was in excellent agreement with experimental results. In particular, it was shown that the interaction between the conductor and the driver coil could be understood and represented as a complex frequency-dependent self-inductance coefficient. Previous works make mention of complex inductances [18], but this phenomenon was not clearly described in terms of electromagnetic processes. The analytical expression for this complex coefficient, which accounts for real (inductive) and imaginary (resistive) elements associated with the rod, fell out of the theory naturally. Thus, electromagnetic field theory and circuit theory have been intuitively combined to provide a complete model of eddy current induction phenomena.

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In this work, the theory developed in [16,17] is extended to include a pickup coil. The coupled circuit equations, which describe the currents flowing in the driver and pickup, are formulated in terms of a conducting and ferromagnetic rod's impulse response using convolution theory, and solved by Fourier transform. The resulting solutions account for the self-coupling of the driver and pickup coils through the sample (complex self-inductance), and for the mutual coupling of the coils through the sample (complex mutual inductance). All electromagnetic coupling and feedback effects are, therefore, incorporated into the model. Experimental results, obtained for the case of square wave excitation, are in excellent agreement with the analytical equations. The exact time-harmonic solutions, for applications in conventional eddy current testing, are also given.

2. Theory

A description of the model geometry is as follows. A pickup coil is centered about the axis of a larger encircling driver coil, as shown in Fig. 1, where $b_1 > a_1 > b_2 > a_2$. These coaxial coils are centered about the axis of a long, ferromagnetic and conducting rod.

In accordance with Maxwell's equations, time-varying currents flowing in the driver and pickup coils will induce eddy currents within the volume of the rod. These eddy currents give rise to transient magnetic fields which, in turn, induce currents within the coils. The circuit equations describing the resultant time-dependent currents $i_1(t)$ and $i_2(t)$ flowing in the driver and pickup, respectively, are written using Kirchhoff's laws in the following general form

$$R_1 i_1(t) = v(t) + \varepsilon_1(t)\,, \tag{1}$$

$$R_2 i_2(t) = \varepsilon_2(t)\,, \tag{2}$$

where $v(t)$ is any time-dependent excitation voltage - step, harmonic, multi-frequency, ramp, saw-tooth, etc. - R_1 and R_2 are the total circuit resistances, and $\varepsilon_1(t)$ and $\varepsilon_2(t)$ are the total time-dependent voltages induced in the driver and pickup coils, respectively. Both $\varepsilon_1(t)$ and $\varepsilon_2(t)$ have three components; one arising from the field generated by the driver coil, another from the transient field generated by the receiver coil and the third from transient eddy current and magnetization fields emanating from a ferromagnetic conducting sample.

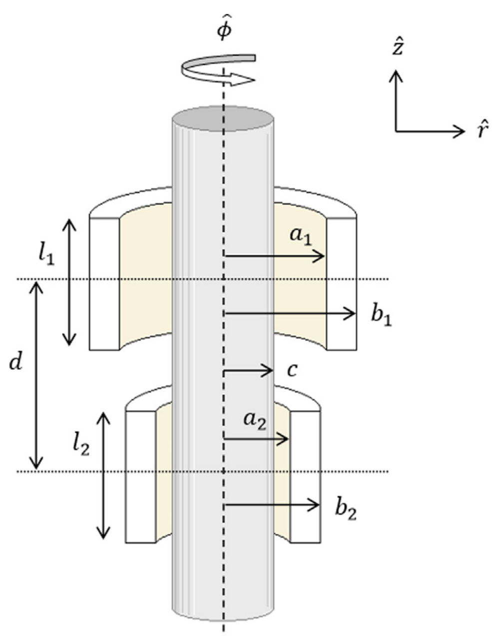

Fig. 1. Coaxial driver coil, pickup coil and rod configuration.

Following the steps in [16,17], induced voltages $\varepsilon_1(t)$ and $\varepsilon_2(t)$ are expressed as the convolution of the system's impulse response with the as yet unknown time-dependent current functions $i_1(t)$ and $i_2(t)$ such that

$$\varepsilon_1(t) = -\frac{2\pi N_1}{l_1(b_1-a_1)}\frac{\mathrm{d}}{\mathrm{d}t}\oint_{S_1} r\Big(\Big(\hat{\psi}_1(r,z,t)+\hat{\Xi}_1(r,z,t)\Big)*i_1(t) + \Big(\hat{\psi}_2(r,z,t)+\hat{\Xi}_2(r,z,t)\Big)*i_2(t)\Big)\mathrm{d}r\mathrm{d}z\,, \tag{3}$$

$$\varepsilon_2(t) = -\frac{2\pi N_2}{l_2(b_2-a_2)}\frac{\mathrm{d}}{\mathrm{d}t}\oint_{S_2} r\Big(\Big(\hat{\psi}_1(r,z,t)+\hat{\Xi}_1(r,z,t)\Big)*i_1(t) + \Big(\hat{\psi}_2(r,z,t)+\hat{\Xi}_2(r,z,t)\Big)*i_2(t)\Big)\mathrm{d}r\mathrm{d}z \tag{4}$$

where $\hat{\psi}(r,z,t) = \hat{\psi}(r,z)\delta(t)$ corresponds to the magnetic vector potential generated by a coil carrying an impulse current, and $\hat{\Xi}(r,z,t)$ is the impulse response of the conducting structure in the air region surrounding the rod. The number in the subscript of $\hat{\psi}$ and $\hat{\Xi}$ specifies the coil from which the potentials arise, whereas S_1 and S_2 specify the coil cross-section over which the potential is integrated. Eqs. (3) and (4) are substituted into the circuit equations in (1) and (2), to which a Fourier transform, defined as $\mathscr{F}\{f(t)\} \equiv \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\mathrm{d}t$ where ω is angular frequency, is applied giving

$$\mathrm{R}_1 I_1(\omega) = V(\omega) - j\omega\frac{2\pi N_1}{l_1(b_1-a_1)}\oint_{S_1} r\Big(\Big(\hat{\psi}_1(r,z)+\hat{\Xi}_1(r,z,\omega)\Big)I_1(\omega) + \Big(\hat{\psi}_2(r,z)+\hat{\Xi}_2(r,z,\omega)\Big)I_2(\omega)\Big)\mathrm{d}r\mathrm{d}z\,, \tag{5}$$

$$\mathrm{R}_2 I_2(\omega) = -j\omega\frac{2\pi N_2}{l_2(b_2-a_2)}\oint_{S_2} r\Big(\Big(\hat{\psi}_1(r,z)+\hat{\Xi}_1(r,z,\omega)\Big)I_1(\omega) + \Big(\hat{\psi}_2(r,z)+\hat{\Xi}_2(r,z,\omega)\Big)I_2(\omega)\Big)\mathrm{d}r\mathrm{d}z\,, \tag{6}$$

where $I(\omega) \equiv \mathscr{F}\{\, i(t)\}$. Thus, transformed current functions $I_1(\omega)$ and $I_2(\omega)$ can be removed from the integrals and the coupled equations can be solved. For conciseness, and in anticipation of the final result, the following definitions are made

$$L_1 \equiv \frac{2\pi N_1}{l_1(b_1-a_1)}\int_{-l_1/2}^{l_1/2}\int_{a_1}^{b_1} r\hat{\psi}_1(r,z)\mathrm{d}r\mathrm{d}z, \tag{7}$$

$$L_2 \equiv \frac{2\pi N_2}{l_2(b_2-a_2)}\int_{-l_2/2}^{l_2/2}\int_{a_2}^{b_2} r\hat{\psi}_2(r,z)\mathrm{d}r\mathrm{d}z\,, \tag{8}$$

$$M \equiv \frac{2\pi N_1}{l_1(b_1-a_1)}\int_{d-(l_1/2)}^{d+(l_1/2)}\int_{a_1}^{b_1} r\hat{\psi}_2(r,z)\mathrm{d}r\mathrm{d}z = \frac{2\pi N_2}{l_2(b_2-a_2)}\int_{d-(l_2/2)}^{d+(l_2/2)}\int_{a_2}^{b_2} r\hat{\psi}_1(r,z)\mathrm{d}r\mathrm{d}z, \tag{9}$$

$$\mathscr{L}_1 \equiv \frac{2\pi N_1}{l_1(b_1-a_1)}\int_{-l_1/2}^{l_1/2}\int_{a_1}^{b_1} r\hat{\Xi}_1(r,z,\omega)\mathrm{d}r\mathrm{d}z\,, \tag{10}$$

$$\mathscr{L}_2 \equiv \frac{2\pi N_2}{l_2(b_2-a_2)}\int_{-l_2/2}^{l_2/2}\int_{a_2}^{b_2} r\hat{\Xi}_2(r,z,\omega)\mathrm{d}r\mathrm{d}z\,, \tag{11}$$

$$\mathscr{M} \equiv \frac{2\pi N_1}{l_1(b_1-a_1)}\int_{d-(l_1/2)}^{d+(l_1/2)}\int_{a_1}^{b_1} r\hat{\Xi}_2(r,z,\omega)\mathrm{d}r\mathrm{d}z = \frac{2\pi N_2}{l_2(b_2-a_2)}\int_{d-(l_2/2)}^{d+(l_2/2)}\int_{a_2}^{b_2} r\hat{\Xi}_1(r,z,\omega)\mathrm{d}r\mathrm{d}z\,, \tag{12}$$

where L_1 and L_2 are the self-inductance coefficients of the driver and pickup coils respectively and M is the mutual inductance coefficient. The limits of integration correspond to the dimensions and relative positions of the coils shown in Fig. 1. Expressions for Eqs. (7)–(9) have been developed previously in [16] and are given here as

$$L_1 = \frac{8\mu_0 N_1^2}{l_1^2(b_1-a_1)^2}\int_0^{\infty}\int_0^{\infty}\frac{\gamma\Big(\int_{a_1}^{b_1} rJ_1(\gamma r)\mathrm{d}r\Big)^2\sin^2\Big(\frac{\lambda l_1}{2}\Big)}{\lambda^2\Big(\gamma^2+\lambda^2\Big)}\mathrm{d}\gamma\mathrm{d}\lambda\,, \tag{13}$$