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Transient response of a driver coil in transient eddy current testing

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ABSTRACT

A solution is presented for the case of a driver coil encircling a ferromagnetic conducting rod. The differential circuit equation is formulated in terms of the rod's impulse response using convolution theory, and solved by Fourier transform. The final solution accounts for feedback between the ferromagnetic rod and the driver coil, providing correct voltage response of the coil. Also arising from the solution is an analytical expression for the complex inductance in the circuit, which accounts for real (inductive) and imaginary (loss) elements associated with the rod. Experimental results, obtained for the case of square wave excitation, show excellent agreement.

1. Introduction

Research in eddy current theory has grown considerably in recent years due to increasing demands for magnetically sensitive non-destructive testing capabilities. Aging nuclear and petrochemical facilities, as well as aircraft and naval fleets, which need to be monitored and maintained, are examples of the benefactors of such technologies. Much work [1–13] has been devoted to the development of theoretical models, which aim to predict induced voltages or impedance changes in interrogating coils for applications in eddy current testing. To date, analytical models that predict coil impedance change as a function of an applied sinusoidal current frequency have been very successful. By contrast, transient eddy current models have been developed on the assumption of an invariant current excitation, as was done in [14–16] for example, which is problematic, since changing inspection conditions and variations in material characteristics modify the applied current signal via magnetic coupling effects. A limited number solutions predicting voltage transients have been developed, and achieving agreement between theory and experiment remains a challenge, particularly at early times and when considering ferromagnetic materials [11]. Ferromagnetic conductors, such as steel, exhibit stronger and thereby more complicated feedback effects between driver and sample circuit elements. Since steel is a commonly encountered construction material, complete models, which correctly account for these complex electromagnetic interactions, are of significant interest. This work considers a coil's voltage response to an abruptly applied voltage step, instead of a current step. Unlike current, a prescribed excitation voltage will remain invariant under changing inspection conditions. Ultimately, model-assisted analyses of experimental data may provide a direct method for the extraction of values such as liftoff, wall thickness, conductivity, permeability and other material and geometrical characteristics of interest.

Previously [17], an approach, in which differential circuit equations were formulated in terms of the magnetic fields arising in an inductive system, was developed and applied for the simple case of a pair of coaxial coils. Expressions, which corresponded to the self- and mutual inductance coefficients, arose naturally from the theory. In this work, the theory is extended to consider the inductive effects of ferromagnetic and conducting structures on a driver coil. In particular, it will be shown that an additional inductance coefficient, which describes the inductive coupling of the coil with a structure, emerges from the theory. Experimental results validate the theory. The resulting analytical model provides a complete understanding of the way in which geometrical and material parameters affect measured coil responses. Furthermore, the theory may be extended to systems containing multiple sensing coils. This will be the focus of subsequent works prepared by the authors.

In following with the theory developed in [17], the impulse response of a long ferromagnetic conducting rod will be directly incorporated into the differential circuit equation via a convolution integral [18], and the resulting equation will be solved by Fourier transform.

2. Theory

A circular driver coil is centred about the axis of a long, ferromagnetic and conducting rod with magnetic permeability μ

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and conductivity σ as depicted in Fig. 1. In accordance with Maxwell's equations, a time-varying current flowing in the driver coil will induce eddy currents within the volume of the rod. These eddy currents give rise to a transient magnetic field which, in turn, generates an opposing current within the driver coil. The magnetic vector potential, arising from the magnetization and induction of eddy currents within a rod following an impulse excitation, is developed first.

The solution for the air region surrounding the rod is the superposition of magnetization and eddy current fields together with the time-dependent excitation field generated by the coil. Consider the case in which the coil carries a current impulse, $i(t) = \delta(t)$, where $\delta(t)$ is a Dirac delta function [18]. Then, the system impulse response outside of the rod, defined as $\hat{A}_{\text{air}}(r,z,t)$, is the superposition of an impulse excitation, denoted $\hat{\psi}(r,z,t)$, together with the impulse response of the rod, defined as $\hat{\Xi}(r,z,t)$, according to

$$\hat{A}_{\text{air}}(r,\lambda,t) = \hat{\psi}(r,\lambda,t) + \hat{\Xi}(r,\lambda,t). \tag{1}$$

The impulse excitation field has been developed in [13] and is given here as

$$\hat{\psi}(r,\lambda,t) = \psi(r,\lambda)\delta(t) = 2\mu_0 n \frac{\sin\left(\frac{\lambda l}{2}\right)}{\lambda} \left\{ \begin{array}{ll} \mathrm{I}_1(\lambda r)\int_a^b r\mathrm{K}_1(\lambda r)\mathrm{d}r & r \le a \\ \mathrm{K}_1(\lambda r)\int_a^b r\mathrm{I}_1(\lambda r)\mathrm{d}r & r \ge b \end{array} \right\} \delta(t), \tag{2}$$

where l is the coil's length, a and b are its inner and outer radii, as shown in Fig. 1, $n \equiv N/l(b-a)$ is the coil's turn density, and $\delta(t)$ is the current unit impulse.

The rod's impulse response for the air region, $\hat{\Xi}(r,z,t)$, which describes the magnetic vector potential associated solely with magnetization and eddy current effects, obeys Laplace's equation [19]. The vector Laplacian is recast into its differential operator form for the appropriate cylindrical geometry, and the resulting expression's axial coordinate, z, is separated by means of a Fourier cosine transform [20] with parameter λ such that

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2} - \lambda^2\right)\hat{\Xi}(r,\lambda,t) = 0\,. \tag{3}$$

Eq. (3) is Bessel's modified differential equation. The allowable solution is the first-order modified Bessel function of the second kind written as

$$\hat{\Xi}(r,\lambda,t) = \mathcal{A}(\lambda,t)\mathrm{K}_1(\lambda r), \tag{4}$$

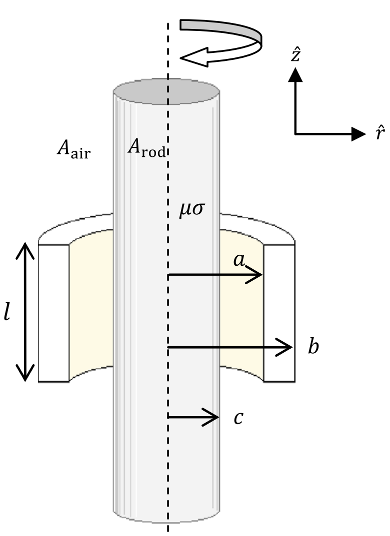

Fig. 1. Diagram of a coil encircling a long rod.

where $\mathcal{A}(\lambda,t)$ is an unknown coefficient. Eqs. (4) and (2) are substituted into (1), and a Fourier transform, defined as $\mathcal{F}\{f(t)\} \equiv \int_{-\infty}^{\infty} f(t)\mathrm{e}^{-j\omega t}\mathrm{d}t$ with angular frequency ω, is applied to the resulting expression given as follows:

$$\hat{A}_{\text{air}}(r,\lambda,\omega) = \psi(r,\lambda) + \mathcal{A}(\lambda,\omega)\mathrm{K}_1(\lambda r), \tag{5}$$

where $\hat{A}_{\text{air}}(r,\lambda,\omega) = \mathcal{F}\left\{\hat{A}_{air}(r,\lambda,t)\right\}$. It is noted that the Fourier transform of $\hat{\psi}(r,\lambda,t)$ is frequency-independent since $\int_{-\infty}^{\infty}\delta(t)\mathrm{e}^{-j\omega t}\mathrm{d}t = 1$.

The diffusion equation [19], which governs the time-dependent evolution of the magnetic vector potential inside the rod, is written as

$$\nabla^2\hat{A}_{\text{rod}}(r,z,t) = \mu_{\text{r}}\mu_0\sigma\frac{\partial}{\partial t}\hat{A}_{\text{rod}}(r,z,t), \tag{6}$$

The z coordinate is separated by means of a Fourier cosine transform and the t coordinate separated by a Fourier transform, such that

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2} - \lambda^2\right)\hat{A}_{\text{rod}}(r,\lambda,\omega) = j\omega\mu_{\text{r}}\mu_0\sigma\,\hat{A}_{\text{rod}}(r,\lambda,\omega). \tag{7}$$

The allowable solution to the differential equation in (7) is the first-order modified Bessel function of the first kind written as

$$\hat{A}_{\text{rod}}(r,\lambda,\omega) = \mathcal{B}(\lambda,\omega)\mathrm{I}_1(\Lambda r), \tag{8}$$

where $\mathcal{B}(\lambda,\omega)$ is an unknown coefficient and Λ is defined as

$$\Lambda \equiv \sqrt{\lambda^2 + j\omega\mu_{\text{r}}\mu_0\sigma}. \tag{9}$$

Exterior and interior solutions (5) and (8), respectively, are matched at the rod's boundary, at which $r = c$ as shown in Fig. 1, using the appropriate boundary conditions [21]

$$\hat{A}_{\text{air}}(c,\lambda,\omega) = \hat{A}_{\text{rod}}(c,\lambda,\omega), \tag{10}$$

$$\mu_{\text{r}}\left(\hat{A}_{\text{air}}(c,\lambda,\omega) + c\hat{A}'_{\text{air}}(c,\lambda,\omega)\right) = \hat{A}_{\text{rod}}(\mathrm{c},\lambda,\omega) + c\hat{A}'_{\text{air}}(\mathrm{c},\lambda,\omega), \tag{11}$$

where the primes denote differentiation with respect to r. Boundary Eqs. (10) and (11) are solved for unknown Bessel coefficients $A(\lambda,\omega)$ and $\mathcal{B}(\lambda,\omega)$:

$$A(\lambda,\omega) = 2\mu_0 n\int_a^b r\mathrm{K}_1(\lambda r)\mathrm{d}r\frac{\sin\left(\frac{\lambda l}{2}\right)}{\lambda}\frac{\mu_r\lambda\mathrm{I}_0(\lambda c)\mathrm{I}_1(\Lambda c) - \Lambda\mathrm{I}_1(\lambda c)\mathrm{I}_0(\Lambda c)}{\mu_r\lambda\mathrm{K}_0(\lambda c)\mathrm{I}_1(\Lambda c) + \Lambda\mathrm{K}_1(\lambda c)\mathrm{I}_0(\Lambda c)}, \tag{12}$$

$$\mathcal{B}(\lambda,\omega) = 2\mu_r\mu_0 n\int_a^b r\mathrm{K}_1(\lambda r)\mathrm{d}r\frac{\sin\left(\frac{\lambda l}{2}\right)(\mathrm{K}_0(\lambda c)\mathrm{I}_1(\lambda c) + \mathrm{I}_0(\lambda c)\mathrm{K}_1(\lambda c))}{\mu_r\lambda\mathrm{K}_0(\lambda c)\mathrm{I}_1(\Lambda c) + \Lambda\mathrm{K}_1(\lambda c)\mathrm{I}_0(\Lambda c)}. \tag{13}$$

Finally, the frequency-domain impulse responses are written as

$$\hat{\Xi}(r,z,\omega) = \frac{1}{\pi}\int_0^\infty A(\lambda,\omega)\mathrm{K}_1(\lambda r)\cos(\lambda z)\mathrm{d}\lambda, \tag{14}$$

$$\hat{A}_{\text{rod}}(r,z,\omega) = \frac{1}{\pi}\int_0^\infty \mathcal{B}(\lambda,\omega)\mathrm{I}_1(\Lambda r)\cos(\lambda z)\mathrm{d}\lambda, \tag{15}$$

where $A(\lambda,\omega)$ and $\mathcal{B}(\lambda,\omega)$ are given in (12) and (13), respectively, and Λ is defined in (9).

In what follows, the rod's impulse response in the region containing the driver coil, Eq. (14), is substituted into the differential circuit equation, as performed previously [17]. The circuit equation, which describes the resultant time-dependent current i flowing in the driver coil, is written using Kirchhoff's laws in the following general form

$$Ri(t) = v(t) + \varepsilon(t), \tag{16}$$

where $v(t)$ is any time-dependent excitation voltage—step, harmonic, multi-frequency, ramp, saw-tooth, etc.—R is the total driver circuit resistance, and $\varepsilon(t)$ is the total time-dependent back-emf induced in the driver coil. The function ε is expected to have two