NDT&E International 61 (2014) 1–9

Contents lists available at ScienceDirect

NDT&E International

journal homepage: www.elsevier.com/locate/ndteint

Probabilistic modeling and sizing of embedded flaws in ultrasonic non-destructive inspections for fatigue damage prognostics and structural integrity assessment

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ARTICLE INFO

Article history:
Received 10 January 2013
Received in revised form
4 September 2013
Accepted 9 September 2013
Available online 17 September 2013

Keywords:
Probability of detection
Non-destructive evaluation
Ultrasonic inspection
Fatigue life prediction
Bayesian
Flaw sizing

ABSTRACT

The paper presents a systematic method and procedure for probabilistic fatigue life prediction using non-destructive testing data under uncertainty. The procedure is developed using uncertainty quantification models for detection, sizing, fatigue model parameters and inputs. The probability of detection model is based on a classical log-linear model coupling the actual flaw size with the NDE reported size. Using probabilistic modeling and Bayes theorem, the distribution of the actual flaw size is derived for both NDE data without flaw indications and NDE data with flaw indications. Fatigue damage and structural integrity assessment are suggested based on the developed method and procedure. A turbine rotor example with realistic NDE inspection data is presented to demonstrate the overall methodology. Calculation and interpretation of the results based on risk recommendations for industrial applications are given. The influence of the NDE detection threshold to the assessment results, and error analysis of the assessment results are discussed in detail.

1. Introduction

Steel and alloy structures are essential parts of civil, aviation, marine, and power generation systems. NDE has been an effective measure to evaluate the manufacturing quality and operation integrity of those structures and systems since the early 1970s [1–3]. Most widely used NDT/E techniques include ultrasonic inspection, magnetic particle inspection, electromagnetic inspection, radiographic inspection, penetrant inspection, acoustic emission, and visual inspection [4–7]. In particular, state-of-the-art ultrasonic NDE techniques provide an opportunity to obtain the information about internal flaws of a structure, such as voids and cracks, without damaging the structure [1]. This information can be integrated with fracture mechanics and material properties, allowing for fatigue life prediction and risk management [8,9].

Scheduled NDEs are sometimes mandatory for structures experiencing time-dependent degradations. Inservice or field inspections are more difficult than inspections in manufacturing phases, and uncertainties in flaw identification and sizing can be much larger due to the more complex conditions for testing [10]. The quality of NDE depends on many uncertain factors, including the sensitivity of inspection instrument, the service condition of the target structure being inspected, the variability of material properties, operation procedure and personnel, and so on. Scientific quantification of these uncertainties must be made in order to produce reliable and informative inspection results. Traditionally, deterministic treatment of the uncertainty uses safety factors [11,12]. The determination of safety factors relies on experiences and expert judgment, which is not a trivial task for normal engineers without strong field knowledge. No universal equations and parameters are available for quantifying uncertainties of a given inspection. Probabilistic methods provide a rational approach for uncertainty management and quantification. However, few studies have been found to provide a complete and systematic procedure for probabilistic modeling and uncertainty quantification in the overall process of ultrasonic NDE-based fatigue life and reliability assessment. The objective of this study is to develop a systematic method for reliable fatigue life prediction using ultrasonic NDE inspection data under uncertainty.

The study is organized as follows. First, probability of detection (POD) modeling is presented using a classical log-linear sizing model to couple the ultrasonic NDE reported flaw size and the actual flaw size. Next, the probabilistic model for actual flaw size is developed. Following that, the overall procedure of probabilistic

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http://dx.doi.org/10.1016/j.ndteint.2013.09.003

fatigue life prediction is proposed. A realistic steam turbine rotor application with ultrasonic NDE data is presented to demonstrate the evaluation for flaw size, fatigue life, and the probability of failure (PoF). Detection threshold of the ultrasonic NDE system and its influence to the assessment result are discussed in detail. Interpretation and error analysis of the assessment result are also presented.

2. Probability of detection modeling

Two approaches are generally available for POD modeling [13,14]. One approach uses hit/miss data, which only record whether a flaw was detected or not. This type of data is still in use for some NDE methods such as penetrant testing or magnetic particle testing. In other NDE inspection systems additional information is available in testing data. For example, the signal amplitude and time index of ultrasonic NDE signals, and the voltage amplitude and location information in electromagnetic responses. In those cases the flaw size or defect severity is closely correlated with signal responses, and thus the NDE data are referred to as signal response data. Signal response data are usually continuous and denoted as $\hat{a}$. The variable of query is usually denoted as a. For example, a can be the actual size of a flaw and $\hat{a}$ is the reported size based on the ultrasonic NDE signal. This study is focused on the signal response data. It has been reported in many studies that $\ln \hat{a}$ and $\ln a$ is usually linearly correlated [15,16] and can be expressed as

$$\ln \hat{a} = \alpha + \beta \ln a + \varepsilon, \tag{1}$$

where ε is a normal random variable with zero mean and standard deviation σ_ε. Both α and β are fitting parameters. A pre-defined threshold $\hat{a}_{\rm th}$ is assumed according to the measurement noise and physical limits of measuring devices. It is also possible that $\hat{a}_{\rm th}$ is specified by manufacturing criterion and standard. For example, a vendor may consider indications less than 1.0 mm are safe to be ignored. A flaw is regarded as identified if $\hat{a}$ exceeds the threshold value of $\hat{a}_{\rm th}$, and the probability of detection of size a can be expressed as

$$\mathrm{POD}(a) = \Pr(\ln \hat{a} > \ln \hat{a}_{\rm th}), \tag{2}$$

where $\Pr(\cdot)$ represents the probability of an event $(\cdot)$. Using Eq. (1), the POD function is rewritten as

$$\mathrm{POD}(a) = \Pr(\alpha + \beta \ln a + \varepsilon > \ln \hat{a}_{\rm th}) = \Phi\left(\frac{\ln a - (\ln \hat{a}_{\rm th} - \alpha)/\beta}{\sigma_\varepsilon/\beta}\right), \tag{3}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function (CDF). If the variable ε follows another probability distribution other than the standard normal distribution, the corresponding CDF of ε should be used.

The log-linear model is one of the most widely used models due to its simple model structure and the property that the flaw size a is ensured to be a positive scalar. Other models, such as a linear model or other physics-based model can also be used to couple the reported flaw size and the actual flaw size. Choosing a particular model format from all available model formats depends on factors such as applications, data characteristics, and inspection systems. From the perspective of model performance, considering model complexity, generality and its predictive power, Bayesian method provides a probabilistic measure for choosing a model based on the concept of Bayes factor [17,18]. Alternatively, any distribution can be used and evaluated by applying the assumption made in Eq. (2) and utilizing Monte Carlo to numerically determine the POD. In fact, even the raw data revealing the correlation between the NDE signal and the true flaw size [19,11] can be used to numerically determine the POD.

3. Probabilistic flaw size modeling under POD

Following the convention, random variables are denoted using capital letters (e.g., $\hat{A}$) and corresponding values are denoted using lower case letters (e.g., $\hat{a}$). Assume that a flaw is detected using NDE and the value of the reported flaw size is a', where a' is a positive real scalar. For convenience, the variable $\ln \hat{A}$ is used instead of $\hat{A}$. Represent probability distributions for propositions $\ln \hat{A} \in (\ln \hat{a},\ \ln \hat{a} + \mathrm{d}\ln \hat{a})$, $\ln A \in (\ln a,\ \ln a + \mathrm{d}\ln a)$, and $\mathcal{E} \in (\varepsilon, \varepsilon + \mathrm{d}\varepsilon)$ by functions $p(\ln \hat{A}) = f_{\ln \hat{A}}(\ln \hat{a})$, $p(\ln A) = f_{\ln A}(\ln a)$, and $p(\mathcal{E}) = f_{\mathcal{E}}(\varepsilon)$, respectively. The probability distribution of the actual flaw size, $p(\ln A)$, is of interest and its derivation is presented below.

Denote D as the event of a flaw is identified and $\overline{D}$ is the event that a flaw is not identified. The joint probability distribution $p(\ln A,\ \ln \hat{A},\ \mathcal{E}|D)$ can be used to obtain $p(\ln A|D)$ as

$$p(\ln A|D) = \int\int p(\ln A,\ \ln \hat{A},\ \mathcal{E}|D)\ \mathrm{d}\ln \hat{A}\ \mathrm{d}\mathcal{E}. \tag{4}$$

The physical meaning of event D represents the fact that an indication of flaw was identified from the NDE inspection data and the resulting reported flaw size is a'. It can be further expressed, under the condition that $\ln \hat{A}$ and $\mathcal{E}$ are independent, as

$$p(\ln A|D) = \int\int p(\ln A|\ln \hat{A}, \mathcal{E}, D) p(\ln \hat{A}|D) p(\mathcal{E}|D)\ \mathrm{d}\ln \hat{A}\ \mathrm{d}\mathcal{E}. \tag{5}$$

Since $\ln \hat{a} = \alpha + \beta \ln a + \varepsilon$,

$$p(\ln A|\ln \hat{A}, \mathcal{E}, D) = \delta(\ln \hat{a} - \alpha - \beta \ln a - \varepsilon), \tag{6}$$

where $\delta(\cdot)$ is the Dirac delta function. Substitute Eq. (6) into Eq. (5) to obtain

$$p(\ln A|D) = \int_R f_{\ln \hat{A}}(\ln \hat{a}) f_{\mathcal{E}}(\ln \hat{a} - \alpha - \beta \ln a)\ \mathrm{d}\ln \hat{a}. \tag{7}$$

The result of Eq. (7) is not final and two cases are discussed as follows.

3.1. Deterministic conversion model

Denote the raw signal feature, such as the maximum echo amplitude in the ultrasonic NDE, as x, and the conversion is made through a mathematical model $m(x)$. It is clear that if the model is perfect $m(x) = \ln \hat{a}$ which leads to

$$f_{\ln \hat{A}}(\ln \hat{a}) = \delta(\ln \hat{a} - m(x)). \tag{8}$$

Substitution of Eq. (8) into Eq. (7) yields (recalling the actual value of $m(x)$ is now $\ln a'$)

$$p(\ln A|D) = \int_R \delta(\ln \hat{a} - m(x)) f_{\mathcal{E}}(\ln \hat{a} - \alpha - \beta \ln a)\ \mathrm{d}\ln \hat{a} = f_{\mathcal{E}}(\ln a' - \alpha - \beta \ln a). \tag{9}$$

Recall $f_{\mathcal{E}}(\cdot)$ is a normal PDF with zero mean and standard deviation σ_ε. It is symmetric and $\alpha + \beta \ln a - \ln a'$ also follows a normal distribution with zero mean and standard deviation σ_ε. Recognize that $\ln A$ follows a normal PDF with mean $(\ln a' - \alpha)/\beta$ and standard deviation σ_ε/β, and thus A is a log-normal variable. The PDF of variable A is

$$p(A|D) = f_{A|D}(a) = \frac{1}{a(\sigma_\varepsilon/\beta)} \phi\left(\frac{\ln a - (\ln a' - \alpha)/\beta}{\sigma_\varepsilon/\beta}\right), \tag{10}$$

where $\phi(\cdot)$ is the standard normal PDF.

3.2. Probabilistic conversion model

If the conversion model is uncertain and the difference between the model output $m(x)$ and the estimated size $\ln \hat{a}$ is a