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Theoretical investigation of nonlinear ultrasonic wave modulation spectroscopy at crack interface

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ABSTRACT

This paper studies theoretical results of a nonlinear ultrasonic method based on interaction of two elastic waves of different frequencies. A virtual Nonlinear Wave Modulation Spectroscopy experiment is performed in the vicinity of a crack described by a model combining classical and hysteretic nonlinearity. Quasistatic response to two frequency excitation was computed and harmonic and intermodulation components were studied. The influence of driving signal parameters and nonlinear parameters on the response is thoroughly discussed. A general way of hysteretic response description based on scaling properties is explained. In case of the combined nonlinear model, an analysis of nonlinear spectral components is performed in complex plane. Based on the complex interaction of classical and hysteretic parts, a method of their separation is proposed.

1. Introduction

In recent years numerous novel ultrasonic methods were proposed which investigate the materials nonlinear elasticity. It was observed experimentally, that the presence of damage significantly influences the nonlinear response of a material. Nonlinear ultrasonic methods were proposed to evaluate properties of inherently nonlinear materials such as composites [1], concrete [2,3], rocks [4–6] and biological tissues [7–10] and also to track an evolution of damage. The exceptional sensitivity to various types of damage was demonstrated on thermal and chemical damage [11], delaminations in composites [1], dispersed microdamage and fatigue cracks [12].

Nonlinear response to dynamic excitation is a consequence of a nonlinear equation of state. Nonlinear stress–strain relations are described various models. The models considered in most nonlinear ultrasonic methods are classical nonlinearity [13], hysteretic nonlinearity [14,15] and Contact Acoustic Nonlinearity (CAN) [16]. The classical nonlinearity is generally attributed to the anharmonicity of interatomic potential and is described by polynomial extension of Hooke's law. The hysteretic constitutive relation arises from mechanics at micro-scale. The Preisach–Mayergoyz space formalism was adapted from the theory of magnetism. It considers a distribution of microcontacts and/or surfaces in friction as possible causes of hysteresis in damaged materials [17–19]. CAN describes contact mechanics at macroscale. CAN uses a bilinear model to describe a step change of stiffness associated with crack opening. In order to achieve opening and closing of crack faces relatively high driving amplitudes are required.

Nonlinear ultrasonic methods use various approaches for characterization of materials nonlinearity. There are spectroscopic methods based on e.g. (sub-) harmonic generation [20], frequency mixing [21,22], resonance frequency shift [23]; and others based on symmetries of nonlinear equation of state, e.g. pulse inversion method (PI) [24] and more general excitation symmetry analysis method (ESAM) [25]; or based on amplitude scaling nonlinearity, i.e. the Scaling Subtraction method (SSM) [2]. Some nonlinear methods were combined with time reversal method [26] for further sensitivity improvement and damage location capabilities [27]. Nonlinear resonant ultrasound spectroscopy (NRUS) and dynamic acoustoelastic testing (DAET) experiments, performed on various materials, enabled more detailed nonlinearity description, particularly the resolution of classical and non-classical nonlinearity [7,28].

This paper provides thorough analysis of two-frequency excitation of a crack described by combined classical nonlinear and hysteretic equation of state. There is a variety of two frequency excitation methods, which were successfully used for crack detection and fatigue process monitoring [21,29–35]. Here we will follow the Nonlinear Wave Modulation Spectroscopy method (NWMS) as proposed in [21,22]. The experiment uses continuous

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two frequency excitation followed by analysis based on amplitude dependence of harmonic and intermodulation (sideband) spectral components. The amplitude of lower (pumping) frequency f_0 is stepwise increased while the amplitude of higher (probing) frequency f_1 is held constant. Due to the continuous excitation, the inspected specimen is in steady state vibration. The volume under inspection by NWMS method can be roughly limited by the position of driving and receiving transducers.

The general approach to the nonlinearity analysis is described in the next section. Then simulated NWMS experiments are performed for classical and hysteretic nonlinearity and for the combined model. Simulation results are presented in the form of amplitude dependences of nonlinear spectral components. Based on analysis results, a new method is proposed, allowing extraction of hysteretic parameter from the response.

2. Concept of nonlinearity analysis

The fundamental concept of our crack response analysis is a quasi-static solution of the nonlinear constitutive relation. Let us consider a single crack surrounded by linear elastic medium. Since the crack is a two-dimensional defect, there is no wave propagation through a nonlinear medium, only localized transmission through crack interfaces. This concept was used in theoretical description of CAN [16]. Experimental evidence suggests that fatigue crack demonstrates both classical [36] as well as hysteretic nonlinearity [21]. The classical nonlinearity is explained by the development of dislocation substructures at crack tip and the hysteresis originates on micro-scale level by contact mechanics of asperities on crack faces. The model proposed in Nonlinear Elastic Wave Spectroscopy (NEWS) methods [21–23] consisting of both classical nonlinear *CN* and hysteretic nonlinear *HN* parts is suitable for crack description. The form from [21–23] was adapted for arbitrary input strain waveforms as

$$d\sigma = K_0\left(1\underbrace{-\beta\varepsilon-\delta\varepsilon^2}_{CN}\underbrace{-\alpha|\varepsilon-\varepsilon_{ep}|}_{HN}\right)d\varepsilon, \tag{1}$$

where K_0 is a linear modulus, β and δ are parameters of quadratic and cubic nonlinearity respectively, α is hysteretic nonlinearity parameter and ε_{ep} is the endpoint amplitude of the current hysteresis loop (note that the endpoint amplitude ε_{ep} is a function of strain history, see Section 3.2 for detailed explanation). The quasi-static solution can be interpreted by a strain wave ε_{in} arriving at the crack interface, where it is transformed by (1) and propagating further in linear medium as ε_{out}. Schematically

$$\varepsilon_{in} \to \sigma_{out} = \int K(\varepsilon_{in})d\varepsilon \to \varepsilon_{out} = \frac{1}{K_0}\sigma_{out}. \tag{2}$$

To follow the methodology proposed in [21] a general review of frequency spectrum properties of ε_{out} is required. We will use the separability of classical and hysteretic parts of the response. The separability holds for the time- and the frequency-domain due to the integration linearity. We can expand (2) as

$$\varepsilon_{out} = \int [1-CN(\varepsilon_{in})-HN(\varepsilon_{in})]\,d\varepsilon = \varepsilon_{in}-\varepsilon^{CN}-\varepsilon^{HN}$$

$$F(\varepsilon_{out}) = F(\varepsilon_{in})-F(\varepsilon^{CN})-F(\varepsilon^{HN}) \tag{3}$$

where separated classical and hysteretic parts of the response are denoted by ε^{CN} and ε^{HN} respectively, F marks Fourier transform. This means that we can perform the analysis of classical and hysteretic response separately, and bring the results together for the analysis of the combined model. Note that (3) holds for complex spectra and quasistatic solution only. In case of dynamic solution linear and nonlinear terms cannot be treated separately. The nonlinearity causes an elastic wave to couple with itself and both nonlinear parts will interact. The analyzed model does not consider attenuation.

Let us now review the parameters entering the analysis. There are parameters of classical nonlinearity β, δ; the hysteretic nonlinearity parameter α and six parameters of the incident wave.

$$\varepsilon_{in} = A_0\ \cos(2\pi f_0 t+\varphi_0)+A_1\ \cos(2\pi f_1 t+\varphi_1). \tag{4}$$

After calculation of the transmitted wave ε_{out} a spectral analysis is performed and harmonic and intermodulation components are extracted. For brevity, characteristics of nonlinear frequencies are denoted by k, l subscript, where k and l is a coefficient of f_0 and f_1 respectively; e.g. the third harmonic amplitude of f_0 is denoted by $A_{3,0}$, the first order sideband phase shift at f_1-f_0 is denoted by $\varphi_{-1,1}$, a complex Fourier component of the second order sideband at f_1+2f_0 is denoted by $Z_{2,1}$, etc.

3. Simulation of nonlinear wave modulation

3.1. Classical nonlinearity

The classical nonlinear part ε^{CN} of the response signal ε_{out} is integrable as

$$\varepsilon^{CN} = \frac{\beta}{2}\varepsilon_{in}^2+\frac{\delta}{3}\varepsilon_{in}^3, \tag{5}$$

where input signal (4) can be directly inserted, and through reduction of trigonometric terms harmonic and intermodulation amplitudes can be separated. The response contains an absolute term A_{00}, second and third harmonics of both frequencies, first and second order sidebands and amplitudes of fundamental frequencies are affected as well. Particular amplitude dependencies are summarized in Table 1. The amplitudes of sidebands are symmetrical. The phase shifts follow the frequency coefficients, in short $\varphi_{k,\,l} = k\varphi_0+l\varphi_1$.

By a comparison of the quasi-static response with the dynamic solution as presented in [21], some common properties can be found. All harmonics and sidebands have the same polynomial degree. The difference is in the proportionality coefficients. Where third harmonic and second order sidebands of quasi-static response depend solely on δ, in case of dynamic response the proportionality parameter is a function of β and δ. The dynamic response contains higher order harmonics and sidebands as well. Properties worth pointing out are that the amplitudes of fundamental frequencies affect each other, while the amplitudes of harmonics remain dependent solely on the amplitude of the corresponding fundamental. The amplitudes of harmonics and sidebands are completely independent of fundamental frequencies values.

Table 1
Overview of harmonic and intermodulation amplitudes of quasi-static response to classical nonlinearity ε^{CN}. Note that intermodulation amplitudes are symmetric i.e. $A_{1,\,1} = A_{-1,\,1}$ and harmonics $A_{0,\,l}$ are analogous to $A_{k,\,0}$ presented in the table.

$A_{0,\,0}$	$A_{1,\,0}$	$A_{2,\,0}$	$A_{3,\,0}$	$A_{1,\,1}$	$A_{2,\,1}$	$A_{1,\,2}$
$\frac{\beta}{4}\left(A_0^2+A_1^2\right)$	$\frac{\delta}{4}A_0^3+\frac{\delta}{2}A_0A_1^2$	$\frac{\beta}{4}A_0^2$	$\frac{\delta}{12}A_0^3$	$\frac{\beta}{2}A_0A_1$	$\frac{\delta}{4}A_0^2A_1$	$\frac{\delta}{4}A_0A_1^2$