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Shape reconstruction of metal pipes with corrosion defects using single frequency limited view scattered data

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ABSTRACT

The inverse problem of reconstructing the cross sectional shape of a metal pipe from single frequency limited view electromagnetic scattered field data is considered. Specifically, the paper addresses the problem of assessing shape changes in the shadow region entailed by limited view data in 2D. The inverse scattering problem is formulated as a non-linear optimization problem that seeks to minimize the difference between measured data and the simulated data through iterations of a forward problem. A modified T-matrix method that exploits the use of FFT to speed up the computations is proposed for solving the forward problem. The proposed methodology is applied for shadow region shape change assessment to determine whether a metal pipe is corroded or not, and to reconstruct the shape of corrosion-like defect, over a range of size parameters. The study is carried out using Transverse Electric (TE) and Transverse Magnetic (TM) polarized fields. Numerical results of inversion using multi-objective optimization over a wide range of size parameter (ka) values show that errors in reconstruction are within 0.5% in the range of $1.4 \leq ka \leq 2.6$. Further, reconstruction with TE is found to lead to better reconstruction than with TM polarization. The effect of random noise in the scattered fields on shape reconstruction is also investigated.

1. Introduction

The inverse scattering problem is to determine the shape and location of the scattering object from measuring the field scattered by the object over some angles (angle diversity) and/or over some frequencies (frequency diversity). This problem has attracted increasing attention owing to its important applications, such as remote sensing, medical imaging, non-destructive evaluation, etc. Two general classes of problems have been considered. The first class deals with a global qualitative or quantitative reconstruction of the internal constitutive object [1–7] and the second class deals with the reconstruction external boundary and localization [8–16]. Problem addressed in this paper belongs to the second class. The general methodologies of the electromagnetic inversion problems can be classified into direct based inversion methods and model based inversion methods. The term direct based inversion means that the electromagnetic properties of the medium are obtained from direct calculations applied to the scattered data. These methods include analytical approximation techniques and layer stripping techniques [17]. Model Based methods can be classified as local and global optimization methods. Local Optimization methods such as quasi-Newton and Gauss–Newton techniques are relatively fast but have the possibility of being trapped in local minima due to the non-linear nature of the problem. For this reason, these techniques are recommended only when sufficient priori information about the inverted model is available. On the other hand, global optimization techniques do not require priori information about the model and for this reason convergence is reached after relatively large number of iterations.

When a monochromatic electromagnetic wave is incident at an angle on an opaque object of arbitrary shape, the object scatters in all directions. The possibility of reconstructing the object's shape depends on the spatial domain over which the scattered field pattern is captured. When angles of incidence and scattering are limited, uncertainties in shape reconstruction arise due to some part of the object would be in the shadow region of the incident field and some part of the object not being in the field of view of the receiver. One of the most widely employed configuration consists of co-located transmitter and receiver employed in applications involving the ground penetrating radar (GPR) . This configuration is the only manner by which objects buried underground are located, sized and imaged. One-sided access renders the problem of shape reconstruction very challenging. The difficulties are compounded by the inhomogeneous

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medium (the Earth) in which the objects are buried. Despite being non-unique and ill-posed, several inverse approaches have been pursued both in the frequency as well as in the time domain. Most microwave inverse scattering algorithms developed are for transverse magnetic (TM) wave illumination in which the vectorial problem is simplified to a scalar one. On the other hand, fewer works have been reported on the more complicated transverse electric (TE) case. In the TE wave excitation case, the presence of polarization charges makes the inverse problem more non-linear. As a result, the reconstruction becomes more difficult. However, the TE polarization case is important because it is found to provide additional information about the object [14]. The effect of frequency on shape reconstruction has been analysed in [15] and found that the shape reconstruction is good in the resonant frequency range for regular shapes without defects in the shadow region. It has been found that the information available from the shadow region decreases if the wave number increases. On the other hand, the scattering pattern becomes isotropic at low frequencies and thus insensitive to the variation of shape [15].

Approach The T-matrix method is a method to solve the direct scattering problems of acoustic, electromagnetic, and elastodynamic waves, where the incident, scattered, and internal fields are expanded in suitable vector wave functions, and the expansion coefficients are determined by enforcing the boundary conditions on the surface of the scatterer. Although considerable amount of literature exists on T-matrix method formulation for 3D scattering, not much is available on 2D scattering. Only recently theoretical formulation of T-matrix method for 2D structures has been presented [18]. Using cylindrical harmonics and Fourier series, a new integral equation formulation is derived for perfectly conducting 2D scattering problems in [19]. A modified T-matrix method was proposed by us in [20] for direct scattering problem, which uses the concept of 2D analogue of T-matrix method to obtain the surface currents and Fast Fourier transforms to speed up the computations.

In Section 2, the theoretical formulation to evaluate the scattered data from a 2D cylinder with arbitrary cross-section using modified T-Matrix method and Cubic spline interpolation techniques is described. In Section 3, the numerical results for the reconstruction of metal pipes with corrosion defects using multi parameter cubic-spline representation are presented. The effects of shadow zone on the reconstruction are discussed and the remedial measures to overcome it are described. Finally, some conclusions are drawn in Section 4.

2. Theoretical formulation

Consider a 2D cylinder of arbitrary cross-section located at a depth d from a line PQ across which measurements are taken. The metal pipe, as shown in Fig. 1, is taken to be a Perfect Electric Conductor (PEC) and the medium is taken to be homogeneous and isotropic defined by relative dielectric constant ϵ_r and relative magnetic permeability μ_r. A line source, denoted by T_x, is positioned at various equi-spaced points along the line PQ to illuminate the cylinder.

2.1. Forward problem

The forward problem is to calculate the scattered fields given the polarization of the incident wave, shape and location of the scatterer. For 2D fields in isotropic media the TM part has only the z component of E field and the TE part has only the z component of H field. Consider PEC object located in homogeneous medium as shown in Fig. 1, the cross-section of the object can be described in polar coordinates in the xy-plane by the equation $\rho = F(\phi)$. The permittivity and permeability of homogeneous medium is denoted by (ϵ,μ).

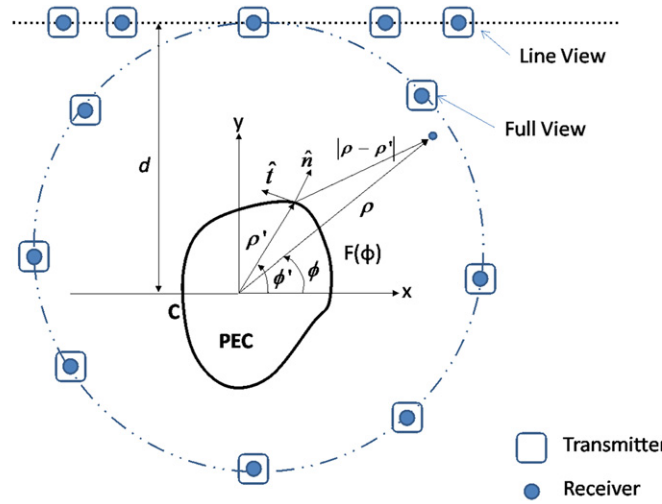

Fig. 1. Geometry for the inverse scattering of a PEC object.

2.1.1. TM polarization

The governing integral equation for TM polarization is derived in Appendix-A. In (A.6) the presence of the term $e^{-jn\phi'}$ inside the integral helps solving this equation very effectively using Fourier series. The current $J_z(\rho')$ is periodic with period 2π and therefore it has a Fourier series of the form:

$$J_z(\rho') = \sum_{m=-\infty}^{\infty} C_m e^{jm\phi'} \tag{1}$$

All the other terms inside the integral (A.6) are expressed by their Fourier series for each value of n.

$$H_n^{(2)}(k\rho')\sqrt{\rho'^2 + \left(\frac{d\phi}{d\rho'}\right)^2} = \sum_{n=-\infty}^{\infty} d_{n,p} e^{jp\phi'} \tag{2}$$

Substituting (1), (2) into (A.6) results in:

$$\int_0^{\pi} \left(\sum_{n=-\infty}^{\infty} C_m e^{jn\phi'}\right)\left(\sum_{n=-\infty}^{\infty} d_{n,p} e^{jp\phi'}\right) e^{-jn\phi'} d\phi' = -I_e H_n^{(2)}(k\rho^s) e^{-jn\rho^s} \tag{3}$$

and after performing the integration:

$$2\pi \sum_{p=-\infty\infty} C_p d_{n,n-p} = -I_e H_n^{(2)}(k\rho^s) e^{-jn\phi^s} \tag{4}$$

Depending on the behavior of the function $\rho' = F(\phi')$ and the size of the problem, only a few number of terms in (4) are required for solving the problem with a reasonable accuracy. Doing so would result in a system of $2M+1$ equations with $2M+1$ unknowns $(C_{-M,}C_M)$

$$2\pi \sum_{p=-M}^{M} C_p d_{n,n-p} = -I_e H_n^{(2)}(k\rho^s) e^{-jn\phi^s} \quad n = -M_{...,}M \tag{5}$$

By using this technique the size of the problem is reduced considerably.

2.1.2. TE polarization

The governing integral equation for TE polarization is derived in Appendix B. Let

$$J_t(\rho') = \sum_{m=-\infty}^{\infty} C_m e^{jm\phi'} \tag{6}$$