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Accurate two-dimensional modelling of piezo-composite array transducer elements

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ABSTRACT

The assumption of piston-like behaviour is widely applied when modelling ultrasonic transducers. Experimental measurements of the directivity patterns of piezo-composite array transducers have shown that this assumption is not valid for small element sizes. An alternative modelling approach has been developed based on the assumption that the variation in pressure across the face of each array element can be described by a Hanning window. The effect of inter-element cross talk has been included in the model by using a window larger than the nominal size of the element. This approach has been shown to produce excellent results via validation against experiment for directivity patterns and array surface displacement.

The improved modelling method has been used to quantify the errors introduced by the assumption of piston-like behaviour by comparison of modelled beam profiles generated using simple delay and sum beam forming. This has been performed by simulating a variety of different beam types and monitoring the following parameters: beam width, maximum amplitude, and beam angle. These simulations show that the only parameter significantly affected is the relative amplitude of different beam angles. The improved directivity model predicts that the maximum beam amplitude decreases with increasing beam angle at a higher rate than directivity models based on piston-like behaviour; the maximum error recorded was approximately 3 dB.

1. Introduction

Ultrasonic phased arrays are now routinely used for non-destructive evaluation (NDE) [1–3], medical diagnosis [4,5] and sonar [6] to generate a variety of different beam types. Due to the complex nature of the ultrasonic fields produced by phased arrays it is common practice to use modelling tools to predict transducer performance.

Ultrasonic field modelling can be performed using a variety of methods. Models based on analytical solutions have been used to model single-element transducers [7,8]. These models are normally based on solutions to the wave equation, such as Kirchhoff theory or the Rayleigh–Sommerfeld theory of diffraction [9]. These models rely on the use of carefully selected Green's functions that satisfy the boundary conditions of the situation being modelled. The resulting integrals must then be solved analytically. This approach can be used for transducer field modelling as well as modelling the scattering of waves from defects [10,11]. The challenge with this approach is that the Green's function selected to model a particular problem may not be suitable for a different scenario, for example changing from a circular transducer to a rectangular shaped transducer. The advantage of this approach is computation speed.

A widely used approach to overcome the limited flexibility of analytical models is to use numerical integration to solve the Rayleigh–Sommerfeld formulation of diffraction, or equivalent formulations [12–15]. This approach is generally referred to as a semi-analytical method, and is commonly used to produce more generic models than a purely analytical approach.

An alternative approach to the semi-analytical method is the edge element approach [16]. This approach is based on dividing the surface of a transducer into a grid of smaller sections, the contribution of each of these can be evaluated analytically by solving a line integral over the perimeter of the section. The full field for the transducer can then be found by the summation of the results from each section. This approach avoids the computationally expensive 2D numerical integration required for methods based on the Rayleigh–Sommerfeld equation, and has been shown to reduce computation times by a factor of 5 [16].

Another method that is commonly used in preference to semi-analytical methods is the use of multi-Gaussian beams. These

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models rely on the summation of a number of Gaussian beams to describe the field produced by a transducer [17,18]. This method is less computationally expensive in comparison to methods that rely on numerical integration, but is less general due to the requirement to determine the expansion coefficients that define the Gaussian beams [18]. The method has been developed for several transducer shapes [19], and with modifications has been shown to accurately represent arrays [20,21].

Regardless of the mathematical framework selected to model an ultrasonic array, an important decision is the selection of an assumed variation in the surface displacement, or pressure, across the face of the array elements. A common approximation used when modelling ultrasonic transducers is the assumption of uniform pressure across the face of the transducer [22], also known as the assumption of piston-like behaviour. When applied to narrow transducers, such as array elements, the accuracy of this assumption has previously been called into question [23,13], but has been shown to be an accurate approximation for larger piezo-composite transducers [24].

In the following section, the accuracy of the assumption of piston-like behaviour will be investigated by comparison of models based on this assumption with experimental results. The experimental results have been collected using a number of modern piezo-composite array transducers, thus enabling the validity of the models to be established over a range of element sizes.

2. Modelling the directivity pattern of array elements

If an array element operating into a liquid is modelled using the Huygens–Fresnel principle of superposition the following directivity function results, assuming that the observation point is in the far field and pressure is uniform across the surface of the element [22]:

$$D(k,\Theta,\alpha) = \text{sinc}\left(\frac{kd_1 \sin\Theta \cos\alpha}{2}\right)\text{sinc}\left(\frac{kd_2 \sin\Theta \sin\alpha}{2}\right) \quad (1)$$

where d_1 is the width and d_2 is the length of the aperture, k is the wave number, and Θ and α are defined in Fig. 1.

If it is assumed that the observation point is located on the x–z plane and $d_2 \gg d_1$ then Eq. (1) can be reduced to the following [22,25–27]:

$$D_f(k,\Theta) = \text{sinc}\left(\frac{kW \sin\Theta}{2}\right) \quad (2)$$

where W is the width of the aperture.

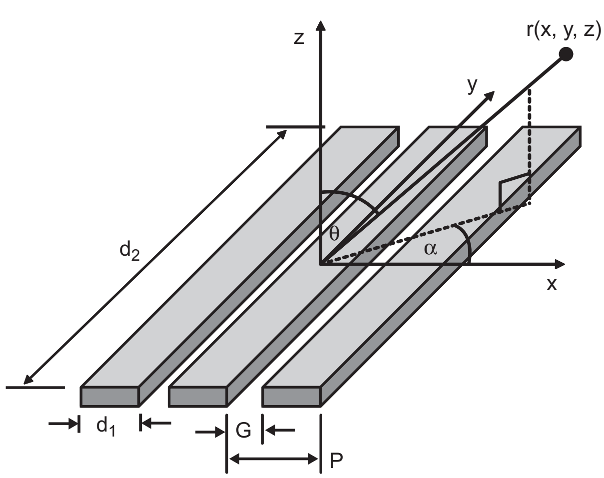

Fig. 1. The coordinate system used to model the array element.

2.1. Ultrasonically measured directivity patterns

The validity of the function expressed in Eq. (2) can be investigated by comparison with experimentally measured directivity patterns of a number of elements from several different array probes. To perform these measurements, a two-axis manipulator has been designed and manufactured that allows all the elements in an array to be measured automatically; the manipulator is shown in Fig. 2. The rotary axis allows a target to be moved around an element at a constant radius, and the linear axis moves the array to allow each element to be measured. Measurements were made by placing the entire rig in a water bath and moving a 2 mm steel rod in a circular path around each element, and recording pulse-echo signals. The manipulator has been designed such that the centre of the circle described by the movement of the target is located on the front face of the array. The directivity pattern is recorded by extracting the maximum amplitude of the first reflection from the rod in each location. The results are normalised by the amplitude of the reflection with the target directly in front of the array. The square root of the values is then taken to convert the combined transmit–receive directivity pattern recorded by the pulse-echo measurement method into a transmit directivity pattern. The directivity pattern of the array elements is assumed to be identical in transmit and receive.

The results from measuring several different commercially available arrays manufactured from 1–3 piezo-composite are presented in Fig. 3, and the specifications of these arrays can be seen in Table 1. A measurement radius of 30 mm was used to ensure that the target is in the far field of the elements. The plotted profiles are the averaged result from the measurement of several different elements in the same array. The mean standard deviation of the results over all the angular positions measured was below 0.01. This demonstrates that for the arrays measured there is very little variation in directivity patterns for elements within the same array; it also shows that the agreement with a Fourier synthesis of Eq. (2) over a frequency range representative of the measured arrays is poor for array types A and B.

The relative sensitivity of each element in the arrays used to generate the results presented in Fig. 3 has also been measured. Relative sensitivity is defined as the combined transmit-receive sensitivity of each element relative to the mean value of the array. The element sensitivity results show that there is typically a variation of less than 2 dB in the relative sensitivity across the arrays, with a small number of outlying elements. This variation does not appear to

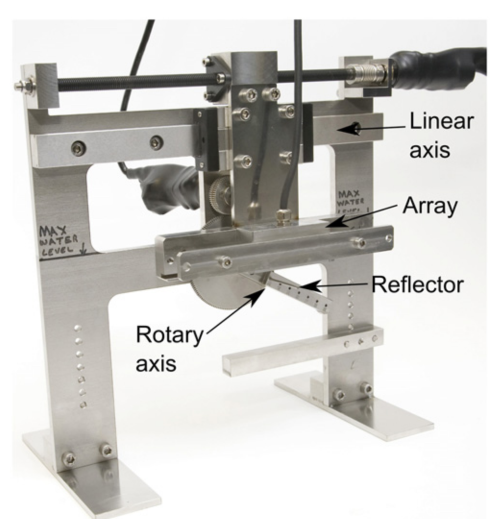

Fig. 2. The manipulator used to measure the directivity patterns of array elements.