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The scattering matrices of Lamb waves at multiple delaminations and broken laminates

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ABSTRACT

Explicit expressions of the scattering matrices of multimodal Lamb waves at multiple horizontal delaminations and broken laminates are derived to characterize the scattering fields when the delamination tips or broken sections keep in alignment vertical to the plate surfaces and mode matching is implemented along the alignment line. The scattering fields of the s_0 mode at a broken laminate with different depths in a plate are specially investigated to evaluate the formulation convergence and effectiveness. It is shown with a proper truncation of the infinite Lamb modes, the calculation results show good convergence to conserve the energy flux and the obtained scattering coefficients are in good accordance with those obtained by numerical simulation.

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1. Introduction

Lamb waves are well-known for their ability to detect defects in plates. Many attempts were made in the past to study the scattering of Lamb waves by various kinds of discontinuities and imperfections such as voids, inclusions and crack-like defects, as well as thickness or material variations [1,2]. Some involve a method called modal decomposition or mode matching method, based on which, the scattering problem could be solved semi-analytically [3–6], thus with clear physical meaning, in comparison with the solutions based on other methods, where to interpret the fields in terms of mode propagation and scattering, either post-processing is often needed, such as the numerical methods like FEM, BEM and the semi-analytical methods called DPSM(distributed point source method) [7], or code-modification in advance is needed, such as the hybrid FEM [8] or BEM [1]. Actually, using the modal decomposition technique, the scattering matrices, which are intuitive quantities to relate the outgoing waves to the incident waves with any modal compositions, respect to a particular scattering region at a particular frequency, could be calculated directly, rather than be investigated case by case by the other methods, depending on the source configurations of the simulation models.

The research on the scattering of Lamb waves at multiple delaminations and broken laminates is mainly inspired by Pagneux's articles on the wave guides with thickness variation [9], where the feasibility of the formulation based on modal decomposition is restricted to tackle the scattering induced by continual and differentiable thickness changes. In comparison, only a few papers handle the scattering problems of Lamb waves near abrupt thickness changes, by the modal decomposition method. Castaings got the solution near a vertical crack with a modal matching based collocation method [4]; Roh solved the fields near a surface notch [10]. however, little research have been devoted to solve the scattering fields of Lamb waves near a singular thickness change by modal matching, noting in the aforementioned two articles, the solutions are obtained only if the boundary conditions of the two opposite step changes are considered simultaneously. One might notice that Demma have solved the scattering of SH waves from a step in a plate [11], however, not that of Lamb waves.

In our previous articles an explicit formula for the reflection matrix of a free plate edge has been derived [12]. Here this formula will be incorporated with Pagneux's formulation [6,9], to solve the scattering problems of multimodal Lamb waves near multiple delaminations or broken laminates cases, which may be encountered in some parallel laminate composites when debonding and matrix cracking occurs [13,14].

The fundamental theory about the modal decomposition technique is introduced in Section 2, which is employed to get the scattering solutions at multiple delaminations in Section 3

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and broken laminates in Section 4. In Section 5, the convergence of the formulation is investigated and the scattering coefficients of the s_0 mode at a broken laminate in a steel plate are compared with those gotten from a numerical method.

2. Modal decomposition related theory and concepts

In Pagneux's formulation one can assemble the four fields in a plane strain plate, corresponding to the two displacement components (u and v) in x and y directions (see Fig. 1) and $s=\sigma_{xx}$, $t=\sigma_{xy}$ with σ the stress tensor, into two pairs of vectors $\boldsymbol{X}=(u,t)^{\mathrm{T}}$ and $\boldsymbol{Y}=(-s,v)^{T}$. The fields can be projected on the right-going Lamb modes with a set of wave numbers k_n satisfying $\mathrm{Im}(k_n)>0$ or group velocity $v_g=\mathrm{d}\omega/\mathrm{d}k_n>0$ when k_n is real:

$$X=(u,t)^T=\sum_{n\in N}a_n(x)X_n(y)=\sum_{n\in N}a_n(x)(U_n(y),T_n(y))^T$$
$$Y=(-s,v)^T=\sum_{n\in N}b_n(x)Y_n(y)=\sum_{n\in N}b_n(x)(-S_n(y),V_n(y))^T, \tag{1}$$

where the mode shape functions, U_n, T_n, S_n and V_n of the right-going mode n, corresponding to u, t, s and v of the whole wave fields, are assembled into two vectors $X_n=(U_n,T_n)^T$ and $Y_n=(-S_n,V_n)^T$. The monochromatic time dependence, with pulsation ω, as $e^{-j\omega\tau}$ ($j=\sqrt{-1}$), is omitted here. Assuming the completeness of Lamb modes, Fraser [15] have proposed a biorthogonal relation which makes the set of vectors X_n and Y_n form a biorthogonal basis:

$$(X_n|Y_m)=\int_{-h}^{h}(-U_nS_m+T_nV_m)\mathrm{d}y=J_n\delta_{nm}, \tag{2}$$

where δ_{nm} is the Kronecker delta symbol, J_n has analytical expression given in Ref. [6]. $(.|.)$ means scalar product integrated along the plate thickness, $2h$. The scalar product of the field vectors X and Y is expressed as:

$$(X|Y)=\int_{-h}^{h}(-us+tv)\mathrm{d}y \tag{3}$$

According to Eqs. (1)–(3), one can extract the modal decomposition coefficients a_m and b_m by:

$$\begin{cases}(X|Y_m)=\sum\limits_{n\in N}a_n(x)(X_n|Y_m)=J_ma_m\\(X_m|Y)=\sum\limits_{n\in N}b_n(x)(X_m|Y_n)=J_mb_m\end{cases}. \tag{4}$$

The sets of coefficients a_m and b_m composes two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, which are linked together by the impedance matrix $\mathbf{Z}$, all as functions of variable x in Pagneux's articles [6,9] where transfer relations of the impedance matrices along x direction in the waveguides are also developed to get the scattering matrices near the material changes [6] as well as the continual and differentiable thickness changes [9]. However, to study the scattering problems at an abrupt thickness change, the concept of the impedance matrix is useless here. We will directly go to the introduction of the scattering matrix.

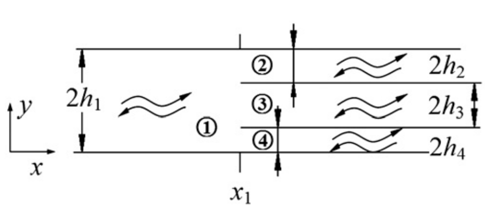

Fig. 1. Scheme graph of a waveguide with double delaminations.

In fact, Eq. (1) is derived from

$$\begin{pmatrix}X\\Y\end{pmatrix}=\sum_{n\in N}A_n(x)\begin{pmatrix}X_n(y)\\Y_n(y)\end{pmatrix}+\sum_{n\in N}B_n(x)\begin{pmatrix}\tilde{X}_n(y)\\\tilde{Y}_n(y)\end{pmatrix}, \tag{5}$$

where the two vectors $\tilde{X}_n(y)$ and $\tilde{Y}_n(y)$, assembled by the mode shapes of the left-going Lamb mode n ($\mathrm{Im}(k_n)<0$ or $v_g<0$ when k_n is real), show symmetry properties corresponding to the vectors $X_n(y)$ and $Y_n(y)$ of the right-going Lamb mode n [6]:

$$\tilde{X}_n(y)=-X_n(y),\quad \tilde{Y}_n(y)=Y_n(y), \tag{6}$$

This makes Eq. (5) become Eq. (1), with

$$a_n(x)=A_n(x)-B_n(x),\quad b_n(x)=A_n(x)+B_n(x), \tag{7}$$

when the sets of $A_n(x)$ and $B_n(x)$, as the modal decomposition amplitudes of the different mode n in both directions of a wave guide at a particular x position, are assembled into two vectors $\boldsymbol{A}$ and $\boldsymbol{B}$, respectively, three matrices can be defined to relates to them at a scattering region: the reflection matrix, the transmission matrix and the scattering matrix. In Section 3, 4, we will drive the scattering matrix of Lamb wave interaction with multiple delaminations and broken laminates, incorporated with the reflection matrix near a free plate edge. Before that, it should be mentioned that usually when modal decomposition is mentioned, it implies the infinite number of Lamb modes are truncated by a large number N to get an approximation. Thus the vectors in this article, if not specially noted, are N dimension ones, and the matrices, $N\times N$ square ones except for the scattering matrices. As a consequence, the convergence of the formulation presented here will be affected by the truncation number N, which will be studied in Section 5.

3. The scattering at multiple delaminations

Considering a structure sketched in Fig. 1, where the plate has two delaminations extending from x_1 to infinite along axis x, we can regard the structure as four semi-infinite plates with a conjunct line at position x_1 along y direction, where the wave fields in the plate i can be donated as $(\boldsymbol{X}_i(x_1,y),\ \boldsymbol{Y}_i(x_1,y))^T$, $(i=1,2,3,4)$ and the corresponding vectors of the Lamb mode n as $(\boldsymbol{X}_{i,n}(x_1,y),\ \boldsymbol{Y}_{i,n}(x_1,y))^{\mathrm{T}}$, the modal decomposition coefficients as $a_{i,n}$ and $b_{i,n}$ according to Eq. (1). The displacement and stress continuities can be expressed as:

$$X_1(x_1,y)=\sum_n a_{1,n}(x_1)X_{1,n}(y)$$
$$=\begin{cases}X_2(x_1,y)=\sum\limits_m a_{2,m}(x_1)X_{2,m}(y) & 2(h_4+h_3)\le y\le 2h_1\\X_3(x_1,y)=\sum\limits_m a_{3,m}(x_1)X_{3,m}(y) & 2h_4\le y\le 2(h_4+h_3)\\X_4(x_1,y)=\sum\limits_m a_{4,m}(x_1)X_{4,m}(y) & 0\le y\le 2h_4,\end{cases} \tag{8a}$$

$$Y_1(x_1,y)=\sum_n b_{1,n}(x_1)Y_{1,n}(y)$$
$$=\begin{cases}Y_2(x_1,y)=\sum\limits_m b_{2,m}(x_1)Y_{2,m}(y) & 2(h_4+h_3)\le y\le 2h_1\\Y_3(x_1,y)=\sum\limits_m b_{3,m}(x_1)Y_{3,m}(y) & 2h_4\le y\le 2(h_4+h_3)\\Y_4(x_1,y)=\sum\limits_m b_{4,m}(x_1)Y_{4,m}(y) & 0\le y\le 2h_4\end{cases}. \tag{8b}$$

To extract $a_{i,m}$ and $b_{i,m}$ in the plate i ($i=1\sim4$), Extraction technique shown in Eq. (4) are employed here in respective

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