NDT&E International 42 (2009) 534–542

Contents lists available at ScienceDirect

NDT&E International

journal homepage: www.elsevier.com/locate/ndteint

Determination of phase spectrum using harmonic wavelet transform

Hyung-Choon Park [a,*], Sung-Eun Joh [b]

[a] Department of Civil Engineering, ChungNam National University, 220 Gung-Dong, Yusung-Gu, Daejeon 305-764, South Korea
[b] Korea Institute of Water and Environment, Korea Water Resources Corporation, 462-1 Jeonmin-Dong, Yusung-Gu, Daejeon 305-730, South Korea

ARTICLE INFO

Article history:
Received 13 January 2008
Received in revised form
24 February 2009
Accepted 30 March 2009
Available online 10 April 2009

Keywords:
Phase difference
Phase spectrum
Harmonic wavelet transform
Dispersion curve

ABSTRACT

The dispersive phase velocity of a wave propagating through a system is an important parameter and carries valuable information in non-destructive tests related to multi-layered systems such as a soil site. The dispersive phase velocity of a wave can be estimated using the phase spectrum, which is easily evaluated through the cross power spectrum. However, the phase spectrum as obtained using the cross power spectrum is sensitive to background noise, which always exists in the field. This causes difficulties in the determination of the dispersive phase velocities. In this paper, a new method to evaluate the phase spectrum using the harmonic wavelet transform is proposed. The introduced method can successfully remove background noise effects. To evaluate the validity of this method, numerical simulations of multi-layered systems were performed. Phase spectra determined by the suggested method were found to be in good agreement with the actual phase spectra under conditions characterized by heavy background noise. This shows the potential of the proposed method.

1. Introduction

Phase velocity of wave propagating through a system is an important parameter and carries valuable information of a system in a non-destructive test. For a multi-layered system such as soil site, the surface wave has dispersive characteristics, implying that the phase velocities vary with the frequency (or the wave length). The frequency–phase velocity curve of a surface wave, which is known as a dispersion curve, is directly related to the geometry and material properties of a system. It is utilized to evaluate the elastic properties and the layer thickness of the site non-destructively [1,2]. To determine a dispersion curve, the phase velocity of any frequency component of wave is determined using the time delay between receivers, as follows:

$$V(f) = D/t(f) \tag{1}$$

Here, D is the distance between two receivers and $t(f)$ is the time delay. The value of $t(f)$ can be obtained for each frequency using the phase difference between signals measured at the two receivers, as follows:

$$t(f) = \phi_{diff}(f)/2\pi f \tag{2}$$

In Eq. (2), $\phi_{diff}(f)$ is the phase difference between two signals at frequency f. Therefore, it is important to determine the correct phase difference for the determination of the dispersion curve in non-destructive site characterizations. The phase difference of each frequency component of wave can be easily calculated using the phase spectrum, which is the phase information of the cross power spectrum via Fourier transform and represents the phase difference between two signals as a function of the frequency [3]. In the field, noise exists continually, and the phase spectrum is easily contaminated by noise owing to the characteristics of the Fourier transform. Noise causes severe contamination of the phase spectrum when the energy of wave is smaller compared to that of the noise in the field. To improve the quality of the phase spectrum under noisy conditions, a larger energy source, such as a vibrosize, should be used. However, this makes the field test somewhat impractical. In addition to the cross power spectrum using the Fourier transform, the time-varying cross power spectra using a time–frequency transform such as a harmonic wavelet can be applied to determine the phase spectrum [4]. The phase spectrum resulting from the use of the time-varying cross spectra is identical to and has the same limitations under noisy conditions as that resulting from the use of the cross power spectrum using the Fourier transform.

Several signal process methods have been developed and applied to remove noise effect [5–7]. These methods can easily remove noise when noise frequency is different from signal frequency. But if signal and noise cover a similar frequency range, it is difficult to remove the noise using these methods. The noise in the field, such as a soil site, is a random noise that generally covers a wide frequency range, including wave signal frequency. In the non-destructive test using a dispersion curve, it is necessary to evaluate the phase spectrum over a wide frequency range [1,2].

* Corresponding author. Tel.: +82 42 821 5673; fax: +82 42 825 0318.
E-mail addresses: civilman@cnu.ac.kr (H.-C. Park), drsecho@hanmail.net (S.-E. Joh).

0963-8695/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ndteint.2009.03.004

Therefore, a new method is needed to determine the reliable phase spectrum even under severe random noise conditions covering a wide frequency range, including signal frequency.

In this paper, a new method to evaluate the phase spectrum under heavy random noise conditions is proposed. It uses the harmonic wavelet transform as an alternative method to the use of the current cross power spectrum method. The harmonic wavelet transform is introduced. The principle of the method and the proposed evaluation procedure of the phase spectrum are described. Finally, numerical simulations of the multi-layered systems are performed and the validity of the proposed method is verified.

2. Harmonic wavelet transform

Wavelet analysis is a fundamental correlation method. The wavelet coefficient, $a(t)$, provides information concerning the structure of the signal and the relationship between the signal and the shape of the analyzing wavelet, $w(t)$. The harmonic wavelet is an orthogonal wavelet represented as follows in the frequency and time domains [8]:

$$\begin{aligned} W_{m,n}(\omega) &= \frac{1}{(n-m)2\pi} \qquad \text{for } m2\pi \leqslant \omega < n2\pi \\ &= 0 \qquad \text{elsewhere} \\ w_{m,n}(t) &= \frac{e^{jn2\pi t} - e^{jm2\pi t}}{j(n-m)2\pi t} \end{aligned} \tag{3}$$

Here, n and m are real but not necessarily integers, and $j = \sqrt{-1}$. Each harmonic wavelet can be related to an ideal bandpass filter as it has a constant real value inside the band of frequency while it is zero elsewhere. In the time domain, the harmonic wavelet has a localized harmonic characteristic.

According to the study of Park and Kim [9], the harmonic wavelet coefficient $a_{m,n}(t)$, which is defined by $W_{m,n}(\omega)$, can be represented as follows:

$$\begin{aligned} a_{m,n}(t) &= s_f(t) + \frac{j}{\pi}\int_{-\infty}^{\infty} \frac{s_f(t')}{t-t'}dt' \\ &= s_f(t) + jH[s_f(t)] = x(t)e^{j\phi_{m,n}(t)} \end{aligned} \tag{4}$$

Here, $s_f(t)$ is the output signal of an ideal bandpass filtering operation in which the magnitude of the filter is $1/2|W_{m,n}(\omega)|$ and its bandpass is $m2\pi \leqslant \omega < n2\pi$. H represents the Hilbert transform, $x(t) = \sqrt{(s_f(t))^2 + (H[s_f(t)])^2}$ is the magnitude of $a_{m,n}(t)$, and $\phi_{m,n}(t) = \tan^{-1}(H[s_f(t)]/s_f(t))$ is the phase of $a_{m,n}(t)$. From Eq. (4), it is apparent that the real part of $a_{m,n}(t)$ is the output signal of the bandpass filtering operation and the imaginary part of $a_{m,n}(t)$ is the Hilbert transform of the real part of $a_{m,n}(t)$; namely, $a_{m,n}(t)$ is the analytic signal corresponding to $s_f(t)$. The output signal of the bandpass filtering operation, $s_f(t)$, is generally an amplitude-modulated signal:

$$s_f(t) = y(t)\cos\phi(t) \tag{5}$$

The analytic signal corresponding to $s_f(t)$ is obtained as follows:

$$\begin{aligned} a_{m,n}(t) = s_f(t) + jH[s_f(t)] &= y(t)\cos\phi(t) + jH[y(t)\cos\phi(t)] \\ &= y(t)\cos\phi(t) + jy(t)\sin\phi(t) \\ &= y(t)e^{j\phi(t)} \end{aligned} \tag{6}$$

Comparing Eqs. (4) and (6), the magnitude of $a_{m,n}(t)$ is shown to represent the envelope of $s_f(t)$ versus the time, and the phase $\phi_{m,n}(t)$ represents an instantaneous phase of $s_f(t)$ versus the time. Through harmonic wavelet transform, magnitude and phase angle time–frequency maps are determined as shown in Fig. 1. These time–frequency maps represent variation of magnitude and phase angle of every frequency component with time.

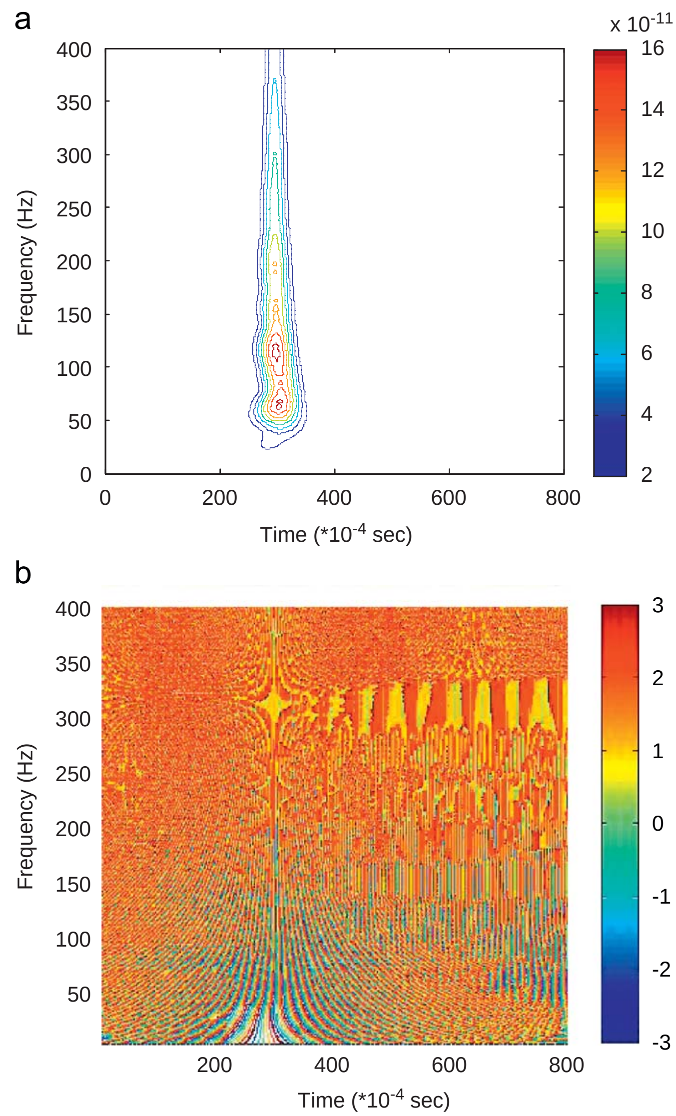

Fig. 1. Harmonic wavelet time–frequency maps of an arbitrary transient wave signal: (a) magnitude time–frequency map; (b) phase angle time–frequency map.

3. Determination of the phase spectrum using the harmonic wavelet transform

3.1. Principle of the method

The phase spectrum represents the phase difference between two time domain signals as a function of frequency. The transient wave signal generated by impact consists of various frequency components, and each frequency component can be represented by an amplitude-modulated signal in the time domain, as shown in Fig. 2(a). A system between two measurement points can be expressed as a bandpass filter. Assuming that the input signal of the system is an amplitude-modulated signal as follows:

$$s_f^1(t) = y_f^1(t)\sin[\phi_f^1(t)] = y_f^1(t)\sin[2\pi ft + \theta^1] \tag{7}$$

The output signal measured at measurement point 2 is also an amplitude-modulated signal with the same frequency,