NDT&E International 42 (2009) 210–214

Contents lists available at ScienceDirect

NDT&E International

journal homepage: www.elsevier.com/locate/ndteint

Time reversal study of ultrasonic waves for anisotropic solids using a Gaussian beam model

Hyunjo Jeong

Division of Mechanical and Automotive Engineering, Wonkwang University, 344-2 Sinyong-dong, Iksan, Jeonbuk 570-749, Republic of Korea

ARTICLE INFO

Article history:
Received 18 April 2008
Received in revised form
21 June 2008
Accepted 29 August 2008
Available online 17 October 2008

Keywords:
Time reversal
Anisotropic materials
Gaussian beam model
Source reconstruction

ABSTRACT

Time reversal (TR) of ultrasonic bulk waves in fluids and isotropic solids has been used in many applications including ultrasonic NDE. However, the study of the TR method for anisotropic materials is not well established. In this paper, the full reconstruction of the input signal is investigated for anisotropic media using an analytical formulation, called a modular Gaussian beam (MGB) model. The time reversal operation of this model in the frequency domain is performed by taking the complex conjugate of the Gaussian amplitude and phase received at the TR mirror position. A narrowband reference signal having a particular frequency and number of cycles is then multiplied and the whole signal is inverse Fourier transformed to obtain the time domain signal. The original input signal is seen to be fully restored by the TR process of MGB model and this model can be more generalized to simulate the spatial and temporal focusing effects due to TR process in anisotropic materials.

1. Introduction

The origin of the time reversal (TR) concept traces back to time reversal acoustics [1–3]. In time reversal acoustics, an input bulk wave can be exactly reconstructed at the source location if a response signal measured at a distinct location is time-reversed and reemitted to the original excitation location. This phenomenon is referred to as TR of bulk waves and has been used in many applications including ultrasonic nondestructive evaluation and underwater acoustics.

While the TR method for bulk waves in fluids and isotropic solids has been well established [4,5], the study of the TR method for anisotropic solids is relatively new. The spatial and temporal focusing effect due to the time reversal mirror (TRM) in anisotropic solid was first studied by Zhang et al. [6,7] by using a ray method. The beam focusing effects will be different depending on the wave propagation direction due to the anisotropy dependence of the time reversal process of propagating waves. Thus, it is necessary to examine whether an original input signal is fully restored at the source location before the TR method is applied for anisotropic media.

In this paper, the full reconstruction of the input signal is attempted through the TR process of ultrasonic bulk waves in anisotropic solids. To achieve this goal, a modular Gaussian beam (MGB) model is employed to simulate the TR process of the longitudinal wave propagation in anisotropic solids. The MGB model provides an efficient formulation for ultrasound propagation, because its properties can be described in analytical matrix form even after propagation through general anisotropic media and after interactions with multiple curved interfaces. It is shown that complete reconstruction of the original input signal can be achieved by the TR process of MGB model.

E-mail address: hjjeong@wonkwang.ac.kr

0963-8695/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ndteint.2008.09.010

2. MGB model for anisotropic solids

We describe a MGB approach for ultrasonic beam propagation shown in Fig. 1, where a single Gaussian beam is radiated from a circular source and travels in solid media composed of two anisotropic solids and an interface. We assume the beam propagation along symmetry directions of anisotropic solids and a normal interface with respect to the beam path. Thus, the x_1–x_3 plane in Fig. 1 constitutes a symmetry plane and the x_3-axis represents one of the symmetry directions. For the geometry of Fig. 1, a Gaussian velocity profile for either a P-, SV- or SH-wave is present at the source and propagates as a Gaussian beam into the solid 1. In Fig. 1, $V_1(0)$ and $\mathbf{M}_1(0)$ are the known starting amplitude and phase values in the Gaussian at the source location ($\tilde{x}_3$). The propagation distance $\tilde{x}_3$ is measured along the central axis of the Gaussian beam, x_3. (x_1,x_2) are coordinates perpendicular to x_3 with x_1 in the plane of incidence and x_2 normal to that plane.

The velocity amplitude and phase of a propagating Gaussian beam in the solid can be completely described by solving the paraxial wave equation (Huang, 2005). For the geometry of Fig. 1, the particle velocity in the Gaussian beam at a distance $x_3 = \tilde{x}_3$

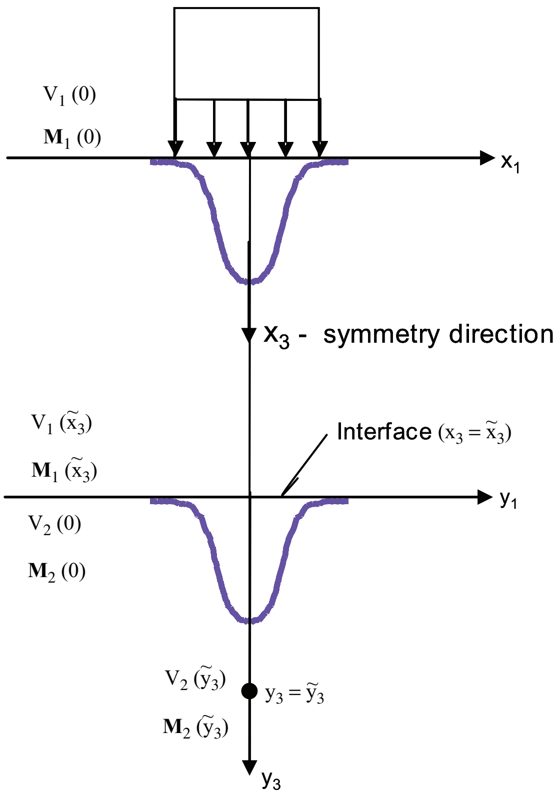

Fig. 1. Propagation of a Gaussian beam in a symmetry direction of anisotropic solid. Two anisotropic solids have a normal interface with respect to the beam path.

can be written as

$$v(\mathbf{x},\omega)=V_1(\tilde{x}_3)\exp\left(\mathrm{i}\omega\left(\frac{\tilde{x}_3}{c_1}+\frac{1}{2}\mathbf{X}^{\mathrm{T}}\mathbf{M}_1(\tilde{x}_3)\mathbf{X}\right)\right) \quad (1)$$

where $\mathbf{X}=[x_1,x_2]^{\mathrm{T}}$, ω the angular frequency, and c_1 is the phase velocity of the particular wave type in the solid 1. It is noticed that the velocity field in Eq. (1) is composed of three terms, i.e., the Gaussian amplitude, the propagation term, and the Gaussian phase. The amplitude $V_1(\tilde{x}_3)$ and phase $\mathbf{M}_1(\tilde{x}_3)$ of a propagating Gaussian beam can be obtained by solving the paraxial wave equation as [8–10]

$$V_1(\tilde{x}_3)=\frac{V_1(0)}{\sqrt{\det[\mathbf{A}_1^P+\mathbf{B}_1^P\mathbf{M}_1(0)]}} \quad (2)$$

$$\mathbf{M}_1(\tilde{x}_3)=[\mathbf{D}_1^{\mathrm{P}}\mathbf{M}_1(0)+\mathbf{C}_1^{\mathrm{P}}][\mathbf{B}_1^{\mathrm{P}}\mathbf{M}_1(0)+\mathbf{A}_1^{\mathrm{P}}]^{-1} \quad (3)$$

where $V_1(0)$ and $\mathbf{M}_1(0)$ are the known starting amplitude and phase values in the Gaussian at the source location ($\tilde{x}_3=0$). $V_1(0)$ and $\mathbf{M}_1(0)$ can be related to the complex constants A_n and B_n of Wen and Breazeale [11] through $V_1(0)=v_0A_n$ and $\mathbf{M}_1(0)=(2\mathrm{i}B_n/\omega a^2)\mathbf{I}$, where v_0 is the uniform velocity on the transducer face, a the radius of a circular transducer, and $\mathbf{I}$ the 2×2 identity matrix. The propagation matrices ($\mathbf{A}_1^p,\mathbf{B}_1^p,\mathbf{C}_1^p,\mathbf{D}_1^p$) are given by

$$\mathbf{A}_1^P=\begin{bmatrix}1&0\\0&1\end{bmatrix},\ \mathbf{B}_1^P=\tilde{x}_3\begin{bmatrix}(c_1-2\tilde{C}_1)&-\tilde{D}_1\\-\tilde{D}_1&(c_1-2\tilde{E}_1)\end{bmatrix},$$
$$\mathbf{C}_1^P=\begin{bmatrix}0&0\\0&0\end{bmatrix},\ \mathbf{D}_1^P=\begin{bmatrix}1&0\\0&1\end{bmatrix} \quad (4)$$

where the terms $(\tilde{C}_1,\tilde{D}_1,\tilde{E}_1)$ represent the slowness surface curvatures of a particular wave type in the solid 1 (as measured in the slowness coordinates (x_1,x_2,x_3)). In the isotropic case, $\tilde{C}_1=\tilde{D}_1=\tilde{E}_1=0$. These curvature terms can be obtained by expanding the x_3 component of the slowness vector, s_3, to the second order in the (x_1,x_2,x_3) coordinates in the form [12,13]

$$s_3=\frac{1}{c_1}+\frac{u_I}{c_1}s_I+K_{IJ}s_Is_J \quad (I,J=1,2) \quad (5)$$

where (u_1,u_2) are the components of the group velocity vector along the (x_1,x_2) axes, respectively, for a wave of type. For an isotropic solid or for a wave propagation in a symmetry direction $u_1=u_2=0$. The matrix $\mathbf{K}$ in Eq. (5) is given by

$$\mathbf{K}=-\frac{1}{2}\begin{bmatrix}c_1-2\tilde{C}_1&-\tilde{D}_1\\-\tilde{D}_1&c_1-2\tilde{E}_1\end{bmatrix} \quad (6)$$

For some simple type of anisotropic media the curvature terms can be expressed in analytical form. In general, they must be obtained numerically from the values of the slowness surfaces in the neighborhood of the refracted ray.

When a particular incident Gaussian beam strikes the interface, reflected and transmitted Gaussian beams are generated. We also assume that the y_1–y_3 plane in Fig. 1 constitutes a symmetry plane and the y_3-axis represents one of the symmetry directions. For the beam propagation along the symmetry direction within the symmetry plane of anisotropic solids and a normal interface with respect to the beam path, there is one transmitted wave of the same type as the incident wave. In order to describe the transmitted wave in solid 2, we employ the coordinates (y_1,y_2,y_3), where $\tilde{y}_3$ is taken along the beam axis y_3.

When the propagated Gaussian beam strikes an interface, reflected and transmitted Gaussian beams are generated. The amplitude $V_2(0)$ and polarization vector $\bar{\mathbf{d}}$ of the particular wave type transmitted in solid 2 at the interface ($\tilde{y}_3=0$) in the paraxial approximation can be found by solving for the problem of the transmission of a plane wave at a planar interface. Thus, the refraction angles of transmitted waves in solid 2 can be determined by Snell's law, and the amplitude $V_2(0)$ can be found by multiplying the incident wave by the appropriate plane wave transmission coefficient. Thus, we have

$$V_2(0)=T_{12}V_1(\tilde{x}_3) \quad (7)$$

where T_{12} is the plane wave transmission coefficient based on the velocity for an incident wave and a transmitted wave. Obtaining the phase at the interface, $\mathbf{M}_2(0)$, is more complicated. It involves matching the phases of the incident and transmitted waves at the interface and approximating the interface surface to the second order (if it is curved) near the point where the central ray of the incident Gaussian strikes the interface.

$$\mathbf{M}_2(0)=[\mathbf{D}_{12}^{\mathrm{t}}\mathbf{M}_1(\tilde{x}_3)+\mathbf{C}_{12}^{\mathrm{t}}][\mathbf{B}_{12}^{\mathrm{t}}\mathbf{M}_1(\tilde{x}_3)+\mathbf{A}_{12}^{\mathrm{t}}]^{-1} \quad (8)$$

where the transmission matrices ($\mathbf{A}_{12}^{\mathrm{t}},\mathbf{B}_{12}^{\mathrm{t}},\mathbf{C}_{12}^{\mathrm{t}},\mathbf{D}_{12}^{\mathrm{t}}$) are given by

$$\mathbf{A}_{12}^{\mathrm{t}}=\begin{bmatrix}1&0\\0&1\end{bmatrix},\ \mathbf{B}_{12}^{\mathrm{t}}=\begin{bmatrix}0&0\\0&0\end{bmatrix},\ \mathbf{C}_{12}^{\mathrm{t}}=\begin{bmatrix}0&0\\0&0\end{bmatrix},\ \mathbf{D}_{12}^{\mathrm{t}}=\begin{bmatrix}1&0\\0&1\end{bmatrix} \quad (9)$$

Similar expressions can be obtained for the interface reflection by modifying the transmission matrices, although not shown here.

In solid 2, the propagation laws for a particular wave type (Gaussian amplitude and phase, $V_2(\tilde{y}_3)$ and $\mathbf{M}_2(\tilde{y}_3)$), follow the same law as in the solid 1.

$$V_2(\tilde{y}_3)=\frac{V_2(0)}{\sqrt{\det[\mathbf{A}_2^{\mathrm{P}}+\mathbf{B}_2^{\mathrm{P}}\mathbf{M}_2(0)]}} \quad (10)$$

$$\mathbf{M}_2(\tilde{y}_3)=[\mathbf{D}_2^{\mathrm{p}}\mathbf{M}_2(0)+\mathbf{C}_2^{\mathrm{p}}][\mathbf{B}_2^{\mathrm{p}}\mathbf{M}_2(0)+\mathbf{A}_2^{\mathrm{p}}]^{-1} \quad (11)$$